This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

45 . 1602

A
COLLECTION OF EXAMPLES
IN
PURE AND MIXED
MATHEMATICS,
WITH
HINTS AND ANSWERS.

BY

ALFRED WRIGLEY, M.A., M.D., F.R.A.S.

AND

WILLIAM H. JOHNSTONE, M.A.

LATE SCHOLARS OF ST. JOHN'S COLLEGE, CAMBRIDGE, AND MATHEMATICAL
MASTERS IN THE HONOURABLE EAST INDIA COMPANY'S
MILITARY SEMINARY, ADDISCOMBE.

LONDON:
LONGMAN, BROWN, GREEN, AND LONGMANS,
PATERNOSTER ROW.
1845.

PRINTED BY RICHARD AND JOHN E. TAYLOR,
RED LION COURT, FLEET STREET.

BIBLIOTHECA BODLEIANA

ADVERTISEMENT.

A DEFICIENCY in Examples illustrative of the different branches of Mathematical Science has long been felt both by Pupils and Tutors. Elementary Works usually contain collections of Examples; but these are generally too limited in number and variety.

The whole course of Mathematics read by Undergraduates in Cambridge will define the scope of the proposed Work. Every subject will be separately illustrated by a copious collection of appropriate Examples. One volume it is hoped will comprise the whole. The present part contains Examples in Arithmetic, Algebra, Logarithms, Geometry, Plane Trigonometry, Mensuration, Conic Sections (or Analytical Geometry of two Dimensions), and such Problems in Mechanics as do not depend for their solution on the Differential and Integral Calculus.

Every problem-question has its answer annexed. In order to ensure accuracy in this particular, every problem solved by one of the Authors has been carefully revised by his colleague. And with the same care has every Theorem in the Work been demonstrated.

With each of the more difficult questions a hint will be given, by the aid of which it is presumed the Student may complete the solution. This will be found especially of service to such Students as are without Tutors.

Those subjects which chiefly occupy the time of Mathematical Students in the Naval and Military Colleges have received especial attention from the Authors. The greatest care has been used in stating the questions so as to render them easily understood. Those problems which in their enunciation and solution bespeak a refined analytical generalization, have been simplified to render them more generally useful, or have been altogether excluded. The chief object has been to supply the greatest number of useful questions solvable by the greatest number of attentive Students.

May 17th, 1845.

TABLE OF CONTENTS.

ARITHMETIC.

	Page		Page
Vulgar Fractions	1.	Answers....	148
Decimal Fractions	5.	——	150
Extraction of Roots	7.	——	151
Duodecimals	9.	——	151
Direct Simple Proportion	9.	——	151
Inverse Simple Proportion	11.	——	152
Compound Proportion	12.	——	152
Practice	13.	——	152
Simple Interest	14.	——	153
Compound Interest	14.	——	153
Discount	15.	——	153
Purchase of Stocks	16.	——	153
Single Fellowship	16.	——	154
Double Fellowship	17.	——	154

ALGEBRA.

Elementary	18.	——	154
Division	19.	——	155
Greatest Common Measure	19.	——	155
Least Common Multiple	20.	——	155
Fractions	20.	——	155
Surds	24.	——	158
Equations	30.	——	161
Problems in Simple Equations	41.	——	171
Problems in Quadratic Equations	43.	——	171

Page Page
Ratio, Proportion, and Variation ...... 45. Answers.... 172
Arithmetical Progression.............. 46. —— 172
Geometrical Progression .... 49. —— 173
Harmonical Progression .... 51. —— 174
Piling of Balls and Shells .... 52. —— 175
Permutations and Combinations........ 53. —— 175
Binomial and Multinomial Theorems .... 55. —— 176
Indeterminate Coefficients .... 58. —— 179
Scales of Notation .... 60. —— 180
Properties of Numbers................ 61. —— 180
Logarithms .... 62. —— 180

GEOMETRICAL DEDUCTIONS, 65.

MENSURATION.

Areas of Plane Figures .... 74. —— 182
Surfaces and Contents of Solids........ 76. —— 183
Measurement of Shot, &c. .... 79. —— 183

TRIGONOMETRY.

Trigonometrical Formulæ .... 81. —— 184
Properties of Plane Figures............ 86. —— 185
Solution of Triangles .... 90. —— 186
Heights and Distances................ 91. —— 187
Expansions, Series, &c. .... 94. —— 187
Solution of Equations .... 96. —— 189
Subsidiary Angles .... 96. —— 190

ANALYTICAL GEOMETRY AND CONIC SECTIONS.

Application of Algebra................ 97. —— *
Straight Line .... 99. —— *
Parabola .... 101. —— *
Ellipse .... 104. —— *
Hyperbola .... 109. —— *
Section of the Cone, &c............... 111. —— *

STATICS.

	Page		Page
Forces on one Point and in the same Plane	116.	Answers....	190
Forces in one Plane, not through the same Point..........................	119.	——	191
Forces not in the same Plane	125.	——	192
The Centre of Gravity................	126.	——	192
Machines	128.	——	193
Friction	132.	——	193

DYNAMICS.

Impact of Bodies....................	134.	——	194
Uniformly accelerated Motion and Gravity	137.	——	195
Motion upon a Curve, and the Simple Pendulum	142.	——	196
Projectiles	143.	——	197

ERRATA.

Page 48, Example 671, *for* a 100 *read* 100.
— 56, —— 790, *for* $\frac{1+x}{1-x}$ *read* $\left(\frac{1+x}{1-x}\right)^n$.
— 63, —— 943, *for* $a^{x^x}+b^{y^y}$ *read* $(a^x)^x \times (b^y)^y$.
— 63, —— 944, *for* $a6^{6x}$ *read* a^{6x}.
— 76, —— 40, *for* semicircular *read* semielliptic.

EXAMPLES

IN

PURE AND MIXED MATHEMATICS.

ARITHMETIC.

VULGAR FRACTIONS.

Reduce the following fractions to their lowest terms.

1. $\frac{60}{144}$, $\frac{96}{324}$, $\frac{420}{5400}$.

2. $\frac{1792}{2048}$, $\frac{6409}{7395}$, $\frac{2433}{13787}$, $\frac{8398}{29393}$.

3. $\frac{23760}{26136}$, $\frac{527751}{645029}$, $\frac{100110}{31866}$, $\frac{1099901}{1067803}$.

4. $\frac{426145749944432}{149720237927424}$, $\frac{7241379310344827586206896551}{9999999999999999999999999999}$.

Reduce the following to simple fractions.

5. $1\frac{11}{12}$, $55\frac{5}{9}$, $1805\frac{3}{8}$, $20\frac{119}{120}$.

6. $3\frac{13}{15}$, $5\frac{11}{17}$, $8\frac{15}{19}$, $9\frac{17}{18}$.

7. $13\frac{20}{21}$, $28\frac{18}{19}$, $100\frac{1190}{1191}$, $16324\frac{3815}{3817}$.

Add together the following fractions.

8. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$.

9. $\frac{2}{5}$, $\frac{35}{80}$, $\frac{14}{100}$, $\frac{3}{140}$, and $\frac{3}{2800}$.

10. $\frac{1}{8}$, $\frac{1}{63}$, $\frac{1}{560}$, and $\frac{1}{5040}$.

11. $8\frac{1}{2}$, $9\frac{2}{3}$, $10\frac{3}{4}$, $11\frac{4}{5}$, and $12\frac{5}{6}$.

12. $387\frac{1}{2}$, $285\frac{1}{4}$, $394\frac{1}{3}$, and 3704.

13. What is the difference between

$$\frac{4}{17}+\frac{7}{50}+\frac{21}{850} \text{ and } \frac{11}{90}?$$

14. Find the value of $\frac{1}{3}+\frac{1}{5}+\frac{6}{7}-\frac{4}{15}$.

15. Find the value of $1-\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$.

16. Find the value of $\frac{15}{16}-\frac{14}{15}+\frac{13}{14}-\frac{11}{12}$.

17. Find the value of $4-\frac{2}{5}-\frac{4}{5}-\frac{3}{4}+\frac{5}{4}-\frac{1}{2}$.

18. Find the value of $1-\frac{2}{3}+\frac{1}{2}+\frac{1}{4}-\frac{7}{6}-\frac{5}{8}+\frac{5}{6}$.

19. Find the product of $\frac{7}{18}$, $\frac{11}{10}$, $\frac{27}{14}$, $\frac{5}{2}$, and $\frac{16}{7}$.

20. Find the product of $\frac{93}{4600}$, and $\frac{125}{662}$.

21. Multiply $\frac{7}{8}$ of $\frac{3}{4}$ by $\frac{2}{5}$ of $\frac{5}{7}$ of 16.

22. Multiply $3\frac{2}{3}$ by $\frac{1}{7}$; and this product by $\frac{3}{5}$ of $\frac{3}{4}$.

23. Find the value of $1+2\times\frac{4}{3}+3\times\frac{16}{9}$.

24. Express the sum of the quantities $1\frac{17}{51}$, $\frac{8}{3}$ of $\frac{41}{34}$, and $\frac{4}{5\frac{1}{10}}$ by a fraction in its lowest terms.

25. What is the difference between $7\frac{4}{5}$ and $7\times\frac{4}{5}$?

26. Simplify the expression $\frac{2}{5}(6\frac{2}{3}+2\frac{1}{2})$.

27. Divide $\frac{22}{35}$ by $\frac{33}{25}$; $3\frac{1}{2}$ by $\frac{1}{2}$; $147\frac{7}{12}$ by $12\frac{3}{5}$; and $15\frac{3}{5}$ by $7\frac{4}{5}$.

28. Divide $\frac{7}{8}$ of 6 by $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{11}{12}$.

29. Find the quotient of $\frac{7}{6}\times\frac{1}{2}\times\frac{3}{10}$ by $\frac{6}{11}\times\frac{7}{8}\times\frac{9}{2}$.

30. Find the quotient of $\frac{1}{7}\times\frac{4}{9}\times 3$ by $\frac{6}{11}\times\frac{8}{9}\times 4$.

31. Find the value of $\frac{2247}{1017} \div \frac{903}{1017} \times \frac{774}{615} \div \frac{1926}{565}$.

32. Simplify $\left(2\frac{1}{2} + \frac{1}{6}\right) \div \left(3\frac{1}{2} - \frac{1}{8}\right)$.

33. Simplify $\frac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$, and $\frac{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}}{1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}}$.

34. Simplify $\frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}}$, and $\frac{\frac{13}{40} - \frac{4}{41}}{\frac{15}{42} - \frac{16}{43}}$.

35. Simplify $\frac{\frac{13}{21} \times \frac{1}{2} - \frac{11}{14} \times \frac{1}{3}}{\frac{16}{21} \times \frac{1}{2} - \frac{13}{14} \times \frac{1}{3}}$, and $\frac{\frac{3}{7} \times 1\frac{2}{5} \times 12\frac{1}{2}}{6\frac{2}{3}}$.

36. Simplify $\frac{\frac{2}{19} + \frac{1}{3}}{3 - \frac{1}{3}} \times \left(\frac{1}{3} + \frac{1}{5}\right)$.

37. Simplify $\frac{18}{17}\left(1 - \frac{64}{81}\right) + \frac{8}{11} \times \frac{1}{6} \times \left(\frac{1}{2} + \frac{5}{12}\right)$.

38. Simplify $\frac{2}{7} \times \frac{1 - \frac{2}{7}}{2} + \frac{4}{5} \times \frac{1}{10} + \frac{3}{5}\left(\frac{1}{2} + \frac{11}{14}\right) + \frac{3}{70}\left(\frac{2}{7} + \frac{4}{5}\right)$.

39. Simplify $\frac{7 - 3\frac{1}{4}}{7 + 3\frac{1}{4}} + \frac{\frac{2}{3}}{\frac{4}{7}} - \frac{1}{2} \times \frac{1}{5} \times \frac{1 + \frac{1}{10}}{1000}$.

40. Reduce $\frac{2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2} - 1}{2\frac{1}{2} \times 2\frac{1}{2} - 1}$.

41. Reduce $\frac{4\frac{1}{3} \times 4\frac{1}{3} \times 4\frac{1}{3} - 4}{4\frac{1}{3} \times 4\frac{1}{3} - 4}$.

42. Reduce $\frac{6\frac{1}{4} \times 6\frac{1}{4} \times 6\frac{1}{4} - 8}{6\frac{1}{4} \times 6\frac{1}{4} - 4}$.

43. Simplify $\frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$, and $\frac{2}{3 + \frac{4}{5 + \frac{6}{7}}}$.

44. Reduce $\frac{5}{7}$ of a penny to the fraction of a pound sterling.

45. What is the value of $\frac{3}{14}$ of half-a-guinea?

46. Find the value of $\frac{4}{5}$ of £3 12*s*.

47. Find the value of $\frac{3}{7}$ of £5 18*s*. 5*d*.

48. Find the value of $\frac{367}{1296}$ of a moidore.

49. Find the value of $\frac{2}{5}$ of $\frac{3}{7}$ of £16 8*s*. $1\frac{1}{2}$*d*.

50. Reduce $\frac{1}{1344}$ of a guinea to the fraction of a farthing.

51. Reduce $\frac{2}{5}$ of 2*s*. $4\frac{1}{2}$*d*. to the fraction of half-a-crown.

52. Reduce $\frac{5}{7}$ of 1*s*. 2*d*. to the fraction of 5*s*. 10*d*.

53. What fraction of half-a-crown is $\frac{3}{5}$ of 6*s*. 8*d*.?

54. Reduce 8*s*. 2*d*. to the fraction of a guinea.
55. Reduce £2 17*s*. $7\frac{1}{5}$*d*. to the fraction of £3 12*s*.
56. What part of a pound is 15*s*. $9\frac{1}{2}$*d*.?

57. Reduce $\frac{3}{7}$ of 1*s*. $5\frac{1}{2}$*d*. to the fraction of half-a-crown.

58. Reduce $\frac{3\frac{1}{9}}{1\frac{2}{13}}\left\{\frac{19}{120}\text{ of £1} - \frac{7}{48}\text{ of 1}s.\right\}$ to the fraction of a moidore.

59. Add together $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ £, $\frac{1}{5}$ of £140 10*s*. 6*d*. and $\frac{3}{5}$ of a guinea.

60. Find the difference between $\frac{1}{19}$ of a pound and $\frac{1}{20}$ of a guinea.

61. Find the difference between $\frac{1}{12}$ £ and $\frac{1}{14}$ of a guinea.

62. Find the values of $\frac{7}{8}$ of a pound sterling, and $\frac{7}{9}$ of a guinea, and reduce their difference to the fraction of a crown.

63. What is the value of $\frac{41}{726}$ of a pound troy?

64. What is the value of $\frac{1}{2}$ of $\frac{6}{5}$ of a pound troy?

65. What is the value of $\frac{63}{160}$ of a ton?

66. Reduce 3 qrs. 3 lbs. 1 oz. $12\frac{4}{9}$ drs. to the fraction of a cwt.

67. Reduce 2 furlongs, 97 yds., 2 ft., 4 in. to the fraction of a mile.

68. Reduce 5 yds. 2 ft. to the fraction of a mile.

69. What is the value of $\frac{5}{8}$ of an acre?

70. What is the value of $\frac{5}{7}$ of $\frac{3}{4}$ of a day?

71. What is the value of $\frac{2}{5}$ day $-\frac{3}{7}$ hour?

72. Reduce 13 w. 5 d. 6 h. 30 m. to the fraction of a year, consisting of $365\frac{1}{4}$ days.

DECIMAL FRACTIONS.

73. Find the sum of ·005, ·0057, and 31·008.

74. Find the sum of 32·394, $7\cdot\dot{4}$, and $\cdot 086\dot{5}\dot{4}$.

75. Subtract 173·45 from 1734·5.

76. Subtract $9\cdot\dot{9}4\dot{6}$ from $27\cdot 04\dot{2}769\dot{4}$.

77. Multiply ·47 by ·0008, and ·000476 by ·0078.

78. Multiply 573·005 by ·000754, and 375·4 by ·057.

79. Divide 1735·5 by 6·5, 6·7288 by 647, and 6 by ·008.

80. Divide 12 by 1728, 16 by 960, and 12·5 by ·0025.

81. Divide 3·1 by ·0025, 2·86 by ·013, and 365 by ·18349.

82. Divide 4·8 by ·00016, ·00001 by ·01, and 3·1 by ·0025.

Reduce the following fractions to decimals.

83. $\frac{2}{25}$, $\frac{1}{800}$, and $\frac{1}{128}$.

84. $\frac{5}{8}$, $\frac{13}{1600}$, $\frac{321}{625}$, and $\frac{56}{1250}$.

85. $\frac{7}{3\frac{1}{5}}$, and $7\frac{1}{2}$ of $\frac{1}{25}+\frac{17}{25}$.

86. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{15}$, $\frac{169}{900}$.

87. $\frac{13}{99}$, $\frac{129}{55}$, $\frac{4111}{33300}$.

Find the fractional values of the following decimals.

88. ·25, ·725, ·1634.

89. 1·36, 3·582, 15·26.

90. ·085, ·00875, ·00023535.

Find the fractions corresponding to the following recurring decimals.

91. $\cdot\dot{2}$, $\cdot\dot{2}\dot{7}$, $\cdot\dot{2}\dot{4}$.

92. $5\cdot 18\dot{7}$, $\cdot 0\dot{5}\dot{1}$, $\cdot 0087\dot{6}\dot{3}$.

93. Divide $\cdot\dot{3}$ by $\cdot\dot{0}\dot{9}$, $\cdot\dot{0}\dot{9}$ by $\cdot\dot{0}\dot{4}$, and $\cdot 042$ by $\cdot 03\dot{6}$.

94. Divide $\cdot\dot{0}\dot{4}$ by $\cdot\dot{7}6923\dot{0}$, 7 by $\cdot\dot{1}4285\dot{7}$, and $234\cdot\dot{6}$ by $\cdot\dot{7}$.

95. Find the value of $2+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\frac{1}{1.2.3.4.5}+\text{\&c.}$

96. Find the value of

$$4\left\{\frac{1}{5}-\frac{1}{3\times 5^3}+\frac{1}{5\times 5^5}-\frac{1}{7\times 5^7}+\text{\&c.}\right\}-\left\{\frac{1}{239}-\frac{1}{3(239)^3}+\frac{1}{5(239)^5}-\text{\&c.}\right\}.$$

97. Find the value of $\dfrac{1}{10+\dfrac{1}{2+\dfrac{1}{30}}}$.

98. Find the value of $2\left\{\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}+\frac{1}{7}\cdot\frac{1}{5^7}+\text{\&c.}\right\}$.

99. Find the value of

$$16\left\{\frac{1}{5}-\frac{1}{3\times 5^3}+\frac{1}{5\times 5^5}-\frac{1}{7\times 5^7}+\text{\&c.}\right\}-\frac{4}{239}.$$

100. Find the value of $2\left\{\frac{1}{3}+\frac{1}{3\times 3^3}+\frac{1}{5\times 3^5}+\frac{1}{7\times 3^7}+\text{\&c.}\right\}$.

101. Find the value of $\dfrac{1}{2\left\{\frac{9}{11}+\frac{1}{3}\left(\frac{9}{11}\right)^3+\frac{1}{5}\left(\frac{9}{11}\right)^5+\text{\&c.}\right\}}$.

102. Reduce 18*s.* 4½*d.* to the decimal of a pound.
103. Reduce 14*s.* 6½*d.* to the decimal of a moidore.
104. Reduce 1½*d.* to the decimal of a pound.
105. Reduce 4*s.* $7\frac{9}{11}$*d.* to the decimal of a pound.
106. Reduce 2*s.* 11¾*d.* to the decimal of a pound.
107. Reduce 8*s.* 7¼*d.* to the decimal of a guinea.
108. Reduce 11 dwts. to the decimal of a pound troy.
109. Reduce 10 drams to the decimal of a pound avoirdupois.
110. Reduce 5 lbs. 10 oz. 3 dwts. 15 grs. troy to the decimal of a cwt. avoirdupois.
111. Reduce 3 qrs. 3 lbs. 1 oz. $12\frac{4}{9}$ drs. to the decimal of a cwt.
112. Reduce 6 furlongs to the decimal of a league.
113. Reduce 12 minutes to the decimal of an hour.
114. Reduce 3 weeks 4 days 5 hrs. 6 min. 7 sec. to the decimal of a month.

115. Reduce 3 weeks 4 days 15 hrs. to the decimal of a year of $365\frac{1}{4}$ days.

116. Reduce $\frac{2}{5}$ of a guinea to the decimal of a pound.

117. Reduce $\frac{4}{5}$ of half-a-guinea to the decimal of a pound.

118. What is the value of ·878125 of a pound sterling?

119. What is the value of ·72708 of a moidore?

120. What is the value of ·375 of a guinea?

121. What is the value of ·75435 of a shilling?

122. What is the value of 1·7962 of a pound sterling?

123. What is the value of ·97216 of a pound sterling?

124. Find the value of ·07 of £2 10*s.*; and express it as the decimal of a pound.

125. What is the value of ·00243 of a pound troy?

126. What is the value of ·0396 of a pound avoirdupois?

127. What is the value of ·475 of a yard?

128. What is the value of ·04535 of a mile?

129. What is the value of ·255 of a league?

130. What is the value of ·05 of an acre?

131. What is the value of $\cdot 089\dot{2}8571\dot{4}$ of 7*s.*?

132. What is the value of $\cdot 013\dot{8}$ of 3·5 moidores?

133. Find the value of £·634375 + ·025 of 25*s.*, $+\cdot 31\dot{6}$ of 30*s.*

134. Find the value of ·75 of 6*s.* 8*d.*, − 1·84375 of 4*s.*, $+3\cdot 979\dot{6}$ of 2*s.*

135. Find the value of $2\cdot 8680\dot{5}$ of 3*s.*, $+\cdot 8\dot{3}$ of 4*s.*, − 1·8 of 5*s.*

136. Find the sum of ·175 of a ton, ·195 cwt., ·145 qr., and ·15 lb.

137. Find the sum of ·573 of an inch, and ·751 of a yard.

138. Find the difference between ·5 of a mile and ·375 of a furlong.

139. Find the value of $\cdot 16\dot{3}$ of $2\frac{1}{2}$ miles.

EXTRACTION OF ROOTS.

140. Find the square roots of 2601, 7225, and 9801.

141. Find the square roots of 390625, 5764801, and 43046721.

142. Find the square roots of 2 and 7658.

143. Find the square roots of 75·347, ·4325, and 876·535.

144. Find the square roots of ·000729, and 36·00000625.

145. Find the square roots of $5\cdot\dot{3}$, $1\cdot\dot{7}$, $6\cdot24\dot{9}$, and $1788\cdot5\dot{7}$.
146. Find the cube roots of 97336, and 405224.
147. Find the cube roots of 941192, and 134217728.
148. Find the cube roots of $\cdot7854$, and $1\cdot092727$.
149. Find the cube roots of $\cdot007077888$, and $\cdot000057464$.
150. Find the fourth roots of 6561, and 1679616.
151. Find the fourth roots of 6724, and $1\cdot2544$.
152. Find the sixth roots of 531441, and 262144.
153. Find the sixth root of 19683.
154. Find the square roots of $\frac{225}{324}$, and $27\frac{9}{16}$.
155. Find the square roots of $29\frac{4}{25}$, and $275\frac{1}{144}$.
156. Find the square roots of $\frac{7}{9}$, $\frac{45}{94}$, and $\frac{448}{507}$.
157. Find the square roots of $\frac{1}{17}$, $\frac{5}{49}$, and $15\frac{5}{8}$.
158. Find the square roots of $8\frac{3}{7}$, $76\frac{14}{17}$, and $\frac{5\cdot04}{\cdot012}$.
159. Find the cube roots of $\frac{324}{150}$, and $5\frac{104}{105}$.
160. Find the cube roots of $\frac{5}{9}$, and $\frac{23}{24}$.
161. Find the values of $\frac{1}{\sqrt{3}}$, and $\frac{1}{2+\sqrt{3}}$.
162. Find the values of $\sqrt{5}+\sqrt{3}$, and $\frac{1}{2}\sqrt{\frac{1}{7}}+\frac{1}{10}$.
163. Find the values of $6+\sqrt{\frac{1}{2}}$, and $\sqrt{\frac{8}{3}}+\sqrt{\frac{3}{8}}$.
164. Find the values of $\frac{1}{2}+\sqrt{\frac{1}{2}}$, and $\sqrt{10}+3$.
165. Find the values of $(\sqrt{2\frac{1}{2}}+\sqrt{3\frac{1}{2}})^2$, and $\left(\sqrt{\frac{3}{8}}-\sqrt{\frac{5}{12}}\right)^2$.
166. Find the values of $\left(\frac{3}{2}\right)^{\frac{3}{2}}$, and $\sqrt[3]{2+\sqrt{3}}$.
167. Find the values of $\frac{3}{2}+\sqrt{3}$, and $\sqrt{\frac{1}{2}}+\sqrt{\frac{2}{7}}$.
168. Find the values of $\sqrt{\frac{3}{8}}-\sqrt{\frac{5}{12}}$, and $(2\sqrt{3}-1)^2$.
169. Find the values of $\left(\sqrt{\frac{8}{3}}+\sqrt{\frac{3}{8}}\right)^2$, and $\left(6-\sqrt{\frac{1}{2}}\right)^2$.
170. Find the values of $(1\cdot1-\sqrt{\cdot1})^2$, and $(\sqrt{\cdot6}-\sqrt{\cdot7})^2$.

DUODECIMALS.

171. Multiply 12 ft. 4 in. by 3 ft. 5 in.
172. Multiply 17 ft. 10 in. by 24 ft. 6 in.
173. Multiply 15 ft. 7 in. by 5 ft. 11 in.
174. Multiply 207 ft. 9 in. by 7 ft. 10 in.
175. Multiply 6 ft. 5 in. 4 pts. by 4 ft. 5 in.
176. Multiply 13 ft. 7 in. 11 pts. by 7 ft. 10 in. 4 pts.
177. Multiply 125 ft. 9 in. 8 pts. by 25 ft. 10 in. 3 pts.
178. Multiply together 3 ft. 4 in., 4 ft. 5 in., and 5 ft. 6 in.
179. Multiply together 23 ft. 5 in. 6 pts., 10 ft. 6 in., and 5 ft. 7 in.

DIRECT SIMPLE PROPORTION.

180. What must I give for 10 pieces of cloth, each piece containing 25 yards, at 17*s.* a yard?

181. How long will a person be in saving £150 at 6*s.* a week?

182. What will 2 cwt. 3 qrs. 4 lbs. of raisins come to at £3 10*s.* per cwt.?

183. How many dozen pairs of gloves, at 2*s.* a pair, will pay for 24 dozen pairs of stockings at 5*s.* a pair?

184. What is the income tax on a rent of £700, at the rate of 7*d.* in the pound?

185. A bankrupt owes me £360 16*s.* 3*d.*, for which I get only £240 15*s.* 6*d.*; how much in the pound is that?

186. If 14*s.* will buy 8 lbs. of tobacco, how much will £4 19*s.* 1½*d.* buy at the same rate?

187. A man bought 120 apples at 3 for a penny, and 120 more at 2 for a penny, and sold the whole at 5 for twopence; did he gain or lose?

188. Having bought 100 pears and 150 apples at the rate of 5 for twopence for the whole, and sold the pears at 2 a penny, and the apples at 3 a penny; do I gain or lose?

189. If 1 lb. of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons; what is the value of a pound of tea when a lemon is worth a penny?

190. I bought 60 yards of cloth at the rate of 5 yards for a guinea, and 70 yards more at the rate of 7 yards for a guinea, and then sold the whole at the rate of 12 yards for 2 guineas; did I gain or lose?

191. A hare pursued by a dog was 86 yards before him at starting; now the dog runs 7 yards for every 5 that the hare runs; how far will the dog have to run before he catches the hare?

192. A person spends £76 5*s.* weekly; what must be his daily income, that at the end of 15 years he may lay by £3761 5*s.*?

193. If the carriage of 3 cwt. 1 qr. 18 lbs. of goods come to

£1 18*s.* $5\frac{1}{4}$*d.*; what will be the charge of carrying 1 ton the same distance?

194. If £100 be sufficient to discharge a debt of £104 17*s.* 6*d.* due a year hence; how much ready money will be sufficient to pay a debt of £1000 due at the same date?

195. The whole income of a parish is £5497 13*s.* 4*d.*, and the sum of £152 10*s.* 6*d.* is to be raised by means of a rate; what is that rate in the pound?

196. A person bought a puncheon of rum for £41 14*s.* 6*d.*; to which he added as much water as reduced the prime cost to 5*s.* 6*d.* a gallon; what quantity of water did he put in (the puncheon containing 84 gallons)?

197. If, when wine is 80 guineas a pipe, a company of 60 people spend 20 guineas therein in a certain time; what is the worth of a pipe, when 15 persons more spend 65 guineas in twice the time, drinking at the same rate?

198. A merchant began the world with £10,000; he gained at the rate of £10,000 in 5 years by trading to China; and at the same time he gained at the rate of £10,000 in 8 years by trading to America; but he spent £10,000 every $2\frac{1}{2}$ years by gaming and extravagance. How many years could he go on before becoming bankrupt?

199. If the pound weight of standard gold be coined into £46 14*s.* 6*d.*; what is the value of an ounce of standard gold?

200. If the pound weight of silver be coined into 66 shillings; what is the weight of half-a-crown?

201. How long would a column of men, extending 3000 feet in length, take to march through a street $\frac{3}{4}$ mile long, at the rate of 75 paces in a minute; each pace being $2\frac{3}{4}$ feet?

202. If £$2\frac{1}{2}$ buy $3\frac{1}{3}$ gallons, how much will £$4\frac{1}{7}$ buy?

203. If $3\frac{1}{5}$ acres let for £$10\frac{1}{4}$, how much will $11\frac{1}{3}$ acres let for?

204. How much per cent. is £62 of £75?

205. If $\frac{2}{3}$ of an estate be worth £220, what is the value of $\frac{3}{11}$ of it?

206. An article which cost 3*s.* 6*d.* is sold for 3*s.* $10\frac{1}{2}$*d.*; what is that per cent. profit?

207. A shilling weighs 3 dwts. 15 grs., of which 3 parts out of 40 are alloy, and the rest pure silver. How much per cent is there of alloy; and what is the weight of pure silver?

208. A person enters France with £57 10*s.*, which he converts into French money; how much ought he to receive, when the pound sterling exchanges for 25 francs, 35 centimes (1 franc = 100 centimes)?

209. The same person, having spent $830\frac{1}{2}$ francs, on leaving

France exchanges the remainder of his French money for English; how much should he receive, the exchange being at 25*f.* 50*c.*?

210. The length of $\frac{1}{360}$th part of the earth's circumference is $69\frac{1}{12}$ miles; what is the earth's diameter, supposing that the diameter of a circle $= \frac{7}{22}$ of its circumference?

211. Express a degree ($69\frac{1}{2}$ miles) in French metres; 32 metres being equal to 35 yards.

212. A quadrant of the meridian $=100000565{\cdot}268$ French metres; and one metre $=39{\cdot}37079$ English inches. How many English yards are there in the quadrant?

213. The number of degrees in an arc of a circle which equals radius is $57{\cdot}29578$. Find the number of seconds in it.

214. If 1 French foot $=1{\cdot}0657654$ English feet, andt the polar diameter of the earth $=3261072{\cdot}9$ toises; each toise being 6 French feet; find the polar diameter in English feet.

215. The length of the seconds pendulum at Greenwich is $39{\cdot}1393$ inches; and the force of gravity is measured by the product of (the length of the seconds pendulum) $\times (3{\cdot}14159)^2$. Find the expression for the force of gravity in feet.

216. The true length of the year being $365{\cdot}24224$ days; find to what the error amounts in 4 centuries, by the reckoning of $365\frac{1}{4}$ days to the year.

INVERSE SIMPLE PROPORTION.

217. If I lend a friend £100 for 12 months, on condition of his returning the favour; for how long ought he to lend me £175?

218. If a board be 8 inches broad, what must be its length to contain 12 square feet?

219. If 4 men *or* 6 women can do a piece of work in 20 days; how long will 3 men *and* 5 women take to finish the same work?

220. How many yards of paper $\frac{3}{4}$ yard wide will hang a room 18 feet long, 15 broad, and 10 high; without allowing for doors and windows?

221. A regiment of 1000 men are to be new-clothed; each coat to contain $2\frac{1}{2}$ yards of cloth of $1\frac{1}{4}$ yard wide; and to be lined with shalloon of $\frac{3}{4}$ yard wide; how many yards of shalloon will there be?

222. How many yards of matting $\frac{3}{4}$ yard wide will be sufficient to cover a floor that is $15\frac{1}{2}$ feet broad, and $27\frac{1}{4}$ feet long?

223. If a certain sum pay for the carriage of 100 lbs. for 120 miles; how far ought an $\frac{1}{2}$ cwt. to be carried for the same money?

224. If 14 men can perform a piece of work in 17·75 days; in how many days can 35 men do the same work?

225. If the step of a man be $2\frac{1}{3}$ feet, and of a horse be $2\frac{3}{4}$ feet; how many horses' paces are equal to 50 men's paces?

226. If 1000 men, marching 8 abreast, extend 325 paces, what will be their extent if they march 10 abreast?

227. If a garrison of 1500 men have provisions for 13 months; how long will their provisions last them, if they be increased in number to 2200?

COMPOUND PROPORTION.

228. If 48 men can perform a piece of work in 16 days of 9 hours each; in how many days of 12 hours each can 64 men accomplish the same?

229. How many days of 15 hours each would 60 men take to perform a piece of work in, when 45 men can do the same in 30 days of 12 hours each?

230. If the expense of 6 workmen for 21 weeks amounts to £120; what will be the expense of 14 workmen for 46 weeks at the same rate?

231. What must I pay for the work of 36 men for 7 months, when the wages of 50 men for 12 months amount to £1080?

232. If 10 horses consume 7 bushels 2 pecks in 7 days; in what time will 28 horses consume 3 qrs. 6 bushels, at the same rate?

233. If £$3\frac{1}{2}$ be the wages of 13 men for $7\frac{1}{2}$ days, what will be the wages of 20 men for $15\frac{1}{3}$ days?

234. If $13\frac{1}{2}$ ells of cloth, $\frac{3}{5}$ yard wide, cost $5\frac{1}{2}$ guineas, what will $33\frac{1}{4}$ yards, $\frac{2}{3}$ of an ell wide, come to?

235. If £$\frac{2}{3}$ be worth $\frac{1}{5}$ of a sheep, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, how much must be given for 100 oxen?

236. If 12 oxen be worth 29 sheep, 15 sheep worth 25 hogs, 17 hogs worth 3 loads of wheat, and 8 loads of wheat worth 13 loads of barley; how many loads of barley must be given for 20 oxen?

237. If 12 of A count for 13 of B, 6 of B for 18 of C, and 13 of C for 2 of D; how many of A count for 100 of D?

238. If the pound weight of standard gold, of 22 carats fine, be

worth £46 14*s*. 6*d*., it is required to find the value of the following East Indian gold coins, the weights and fineness (or number of parts of pure gold in 1000 parts of the coin) being as stated below:

Mohur of Bengal, of weight 7 dwts. 23 grs. and fineness 993.
Mohur of Bombay, of weight 7 dwts. 10½ grs. and fineness 953.
Gold Rupee of Bombay, of weight 7 dwts. 11 grs. and fineness 922.
Gold Rupee of Madras, of weight 7 dwts. 12 grs. and fineness 916.
Star Pagoda of Madras, of weight 2 dwts. 4½ grs. and fineness 792.

239. If the pound weight of standard silver be worth 62*s*., of which 222 parts in 240 are pure silver; it is required to find the value of the following East Indian silver coins, their weights and fineness being as below:

Sicca Rupee, of weight 7 dwts. 12 grs. and fineness 979.
Arcott Rupee, of weight 7 dwts. 9 grs. and fineness 941.
Bombay Rupee, of weight 7 dwts. 11 grs. and fineness 926.
Baroch Rupee, of weight 7 dwts. 10 grs. and fineness 883.

240. If 248 men, in 5½ days of 11 hours each, dig a trench of 7 degrees of hardness, 232½ yards long, 3⅔ wide, and 2⅓ deep; in how many days, of 9 hours long, will 24 men dig a trench, of 4 degrees of hardness, 337½ yards long, 5⅗ wide, and 3½ deep?

PRACTICE.

241. Find the value of 2654 at ¼*d*.
242. Find the value of 3895 at ¾*d*.
243. Find the value of 3826 at 3½*d*.
244. Find the value of 59623 at 7¾*d*.
245. Find the value of 2654 at 8½*d*.
246. Find the value of 38940 at 11¾*d*.
247. Find the value of 8765 at 14½*d*.
248. Find the value of 18541 at 1*s*. 8¾*d*.
249. Find the value of 264 at 6*s*.
250. Find the value of 2084 at 9*s*.
251. Find the value of 20563 at 17*s*.
252. Find the value of 43265 at 19*s*.
253. Find the value of 2684 at 3*s*. 6*d*.
254. Find the value of 6285 at 15*s*. 7*d*.
255. Find the value of 1846 at 7*s*. 6½*d*.
256. Find the value of 24503 at 13*s*. 7¾*d*.
257. Find the value of 265 at £3 15*s*.
258. Find the value of 382 at £6 13*s*.
259. Find the value of 123 at £5 13*s*. 6½*d*.
260. Find the value of 356 at £7 15*s*. 7¾*d*.

SIMPLE INTEREST.

If $P=$ principal, $A=$ amount, $I=$ interest,
$r=$ rate of interest on £1 for 1 year, $t=$ number of years,

$$A=P(1+rt),\quad P=\frac{A}{1+rt},\quad r=\frac{A-P}{Pt},\quad t=\frac{A-P}{Pr},\quad I=Prt.$$

261. Find the interest and amount of £340 15*s.* 6*d.* at 4 per cent., in 1 year.

262. Find the interest and amount of £235 14*s.* 4*d.* at 5 per cent., for 3 years.

263. Find the interest and amount of £1643 7*s.* 5½*d.* for 4 years, at 3½ per cent.

264. Find the interest and amount of £2860 16*s.* 9¾*d.* for 5½ years, at 4¼ per cent.

265. Find the interest and amount of £347 10*s.* at 5 per cent. per annum, for 219 days.

266. What principal put out to interest for 7½ years at 5 per cent., would amount to £125 13*s.* 4*d.* ?

267. What principal put out to interest for 6 years at 4 per cent., would amount to £600 ?

268. In what time will £100 amount to £1000, at 5 per cent. ?

269. In how many years, at 3½ per cent., will £125 double itself?

270. In what time, at 4¼ per cent., will £385 10*s.* treble itself?

271. At what rate will £300, in 7 years, amount to £350 ?

272. At what rate will a sum double itself in 30 years ?

COMPOUND INTEREST.

If $P=$ principal, $A=$ amount, $I=$ interest,
$r=$ rate of interest on £1 for 1 year, $t=$ number of years,

$$A=P(1+r)^t.\quad P=\frac{A}{(1+r)^t},\quad r=\sqrt[t]{\frac{A}{P}}-1,\quad t=\frac{\log A-\log P}{\log(1+r)},$$

$$I=A-P=P\{(1+r)^t-1\}.$$

273. What is the compound interest on £256 10*s.* for 4 years, at 5 per cent. per annum ?

274. What is the compound interest on £2643 13*s.* 8*d.* for 2½ years, at 3 per cent. per annum ?

275. What is the compound interest on £1234 15*s.* 6¾*d.* for 10 years, at 4½ per cent. per annum ?

276. What is the compound interest on £244 17*s.* 6½*d.* for 3¾ years, at 2½ per cent. per annum ?

277. What is the compound interest on £832 12*s.* 10½*d.* for 2 years, 7 months and 73 days, at 3¼ per cent. per annum ?

278. What does £845 17*s.* 10*d.* amount to in 5 years, at 3 per cent. per annum, compound interest?

279. Find the several amounts of £660, payable (1) yearly, (2) half-yearly, and (3) quarterly for 6 years, at 5 per cent. per annum.

280. What is the amount of £420 15*s.* in 4 years, at 3 per cent. per annum, payable half-yearly?

281. What is the amount of £2639 16*s.* 3½*d.* in 5 years, at 4 per cent. per annum, compound interest, payable monthly?

282. To how much will one farthing amount in 1844 years, at 5 per cent., compound interest?

283. What sum will amount to £1000 in 10 years, at 5 per cent., compound interest?

284. What sum will amount to £5000 in 500 years at 4 per cent. per annum, compound interest?

285. At what rate per cent. will £300 amount to £500 in 4 years, compound interest?

286. At what rate per cent. will a given sum double itself in 6 years, compound interest?

287. In how many years will £600 amount to £6000, at 5 per cent., compound interest?

288. In how many years will a given sum treble itself, at 3 per cent., compound interest, payable half-yearly?

DISCOUNT.

If A = amount, P = present value, D = discount,
r = rate of interest on £1 for 1 year, t = number of years,

$$P=\frac{A}{1+rt}, \quad D=A-P=\frac{Art}{1+rt}.$$

289. Find the discount on £1000 due 4 years hence, at 5 per cent. per annum, interest.

290. Find the discount on a bill of £750, which has 9 months to run; interest at 4 per cent. per annum.

291. What is the discount on £256 7*s.* 6*d.* due 3 years hence; interest at 3 per cent. per annum?

292. What is the present value of a bill of £324 16*s.* 7½*d.*, due 2 years and 9 months hence; interest at 3½ per cent. per annum?

293. What is the present value of £10,000 due 500 years hence; interest at 4½ per cent. per annum?

294. What is the discount on £275 6*s.* 8*d.*, due 18 months hence; interest at 4½ per cent. per annum?

295. What is the present value of £1000 10*s.*, due 5 years 4 months hence, at 4½ per cent. per annum?

PURCHASE OF STOCKS.

296. If I buy £10,000 into the 3 per cent. Consols at 93¾, how much must I pay for it; and what interest per cent. shall I derive from it?

297. What quantity of 3½ per cent. stock at 94 can I purchase for £500; and what income do I derive from it?

298. What interest per cent. is derived from purchasing into the 3½ per cents at 101½?

299. How much of the 3 per cent. stock at 94⅛ can I purchase for £50,000?

300. What must be paid for an annuity of £500 per annum in the Long Annuities at 15⅕; and what is the rate per cent.?

301. What is the interest on money in the Long Annuities at 15⅛?

302. How much 4 per cent. stock can be purchased by the transfer of £1000 stock from the 3 per cents at 72 to the 4 per cents at 90?

303. What is the purchase of £1743 in the 3 per cent. stock at 82⅞?

304. The 4 per cents being at 82⅛, what must be given for £1000 stock?

305. If I lay out £2000 in the 3 per cent. Consols, when they are at 88½, what income shall I derive from the purchase?

306. If £1000 be laid out in the purchase of 3 per cent. Consols, which are at 81⅝, at what price should the stock be sold to produce a gain of £100?

307. What sum must be paid for £439 12*s.* 6*d.* in the 3½ per cent. stock at 92⅞?

SINGLE FELLOWSHIP.

308. Four persons, A, B, C, D, rent a pasture for £50; A put in 7 cattle; B, 8; C, 9; D, 10: how much should each person pay for his share?

309. A tax of £530 is to be raised from 3 towns, the respective numbers of inhabitants of which are 2500, 3000, and 4200. How much should each pay?

310. A ship is wrecked, whose value is £1500, of which £800 belonged to A, £400 to B, £200 to C, and £100 to D. The insurance office pays them back £1250 of their loss: how much of this should each receive?

311. Three persons whose estates are respectively worth £1000, £755, £645 a year, buy 100 railway shares among them; each buying a number proportional to his estate. How many shares do each buy?

312. A regiment consisting of 6 companies, the number of men in them being 64, 66, 68, 70, 72, 74 respectively, have to send 207 men on garrison duty; how many should each company furnish in order that they may send proportionally to their strength?

DOUBLE FELLOWSHIP.

313. Four merchants, A, B, C, D, trade together; A's stock of £300 was in trade 12 months; B's stock of £330 for 10 months; C's of £375 for 8 months, and D's of £395 for 6 months. They gain £723; how much of this ought each to receive?

314. A company, consisting of 1 captain, 2 lieutenants, 6 sergeants, 10 corporals, and 50 men, storm a fort, and find there a treasure worth £1600, which they divide among them according to their pay and the time of their service. The captain has been in the service for 5 years, and is paid 11*s.* 7*d.* a day; the lieutenants have served for $3\frac{1}{2}$ years, and are paid 6*s.* 6*d.* a day; the sergeants have served for 7 years, and are paid 2*s.* 6*d.* a day; the corporals have served for 4 years, and are paid 1*s.* 6*d.* a day; the men have served 2 years, and are paid 1*s.* a day: what portion of the prize should each receive?

ALGEBRA.

PART I.

1. Find the value of $\frac{a}{b}-\sqrt{\frac{1+a}{1-b}}$, when $a=\frac{1}{4}$ and $b=\frac{1}{5}$.
2. Show that $x^2-\sqrt{x^2-42x+89}=7$ if $x=2$.
3. What is the numerical value of $x+7\sqrt[3]{x}$ when $x=8$?
4. Add together $(a+7)x^2-by^2$ and $(b-7)x^2-7y^2$.
5. Add together $1-\{1-(1-x)\}$, $2x-(3-5x)$, and $2-(-4+5x)$.
6. Add together $(a^2x-ax^2-x^2)$, $(ax-x^2-a^2)$, $(-2ax^2-2a^2x-2a^2)$, $(-3a^2x+3ax^2+3a^2)$, and $4a^2x+2x^2$.
7. Find the sum of $a-x+(2a-3x)-(a-2x)$, $x-2a-(2x-a)-(x-3a)$, $-(a-x)-(2a+x)+(x+a)$, $-(a-2x)-(3a-5x)-(x-a)$, and $-(2-2a)-(3-2x)+(1-x)$.
8. Find the difference between $x^4-2a(a-b)x^2+(a^2+b^2)(a-b)x-a^2b^2$ and $x^4-(a-b)x^3+(a-b)b^2x-b^4$.
9. Multiply a^2+ax+x^2 by a^2-ax+x^2.
10. Multiply $a-b+c-d$ by $a+b-c+d$.
11. Multiply x^3-ax^2+bx-c by x^2-tx+s.
12. Multiply $a^2+b^2+c^2-ab-ac-bc$ by $a+b+c$.
13. Multiply $\frac{5}{2}x^2+3ax-\frac{7a^2}{3}$ by $2x^2-ax-\frac{a^2}{2}$.
14. Multiply $(y-z)x^2+(y^2-z^2)x+2y^2z-2yz^2$ by $y+z-x$.
15. Multiply $ab^2d-2bcd^2-3a^2cd+4abc^2$ by $a+b+c+d$.
16. Multiply $a^m+b^p-2c^n$ by $2a^m-3b$.
17. Find the continued product of $x-3$, $x+4$, $x-5$, $x+6$.
18. Required the coefficient of x^4 in the product of
$$1+\frac{1}{2}x+\frac{1}{3}x^2+\frac{1}{4}x^3+\text{ \&c. by }1-\frac{1}{3}x+\frac{1}{5}x^2-\frac{1}{7}x^3+\text{\&c.}$$
19. Simplify the expression
$$\frac{1}{6}\left\{x(x+1)(x+2)+x(x-1)(x-2)\right\}+\frac{2}{3}(x-1)x(x+1).$$
20. Find the continued product of $(x-1)$, $x+1$, x^2+1, x^4+1, &c., to $n+1$ factors.

DIVISION.

21. Divide $27a^3b^2cfg+36ab^3de^2-9b^4d^3f^2+12dc^3e^2g^4$ by $-18abceg$.
22. Divide $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.
23. Divide $3h^5+16h^4k-33h^3k^2+14h^2k^3$ by h^2+7hk.
24. Divide a^5-243 by $a-3$.
25. Divide $p^2+pq+2pr-2q^2+7qr-3r^2$ by $p-q+3r$.
26. Divide $x^6-140x^4+1050x^3-3101x^2+3990x-1800$ by $x^3-12x^2+47x-60$.
27. Divide $1+2x$ by $1-3x$.
28. Divide $x^4-\frac{19x^2y^2}{6}+\frac{xy^3}{3}+\frac{y^4}{6}$ by $x^2+2xy+\frac{y^2}{3}$.
29. Divide $(x^3-1)a^3-(x^3+x^2-2)a^2+(4x^2+3x+2)a-3(x+1)$ by $(x-1)a^2-(x-1)a+3$.
30. Divide $abx^3+(ac-bd)x^2-(af+cd)x+df$ by $ax-d$.
31. Divide $-\frac{5}{9}a^2+\frac{11}{3}ab-\frac{10}{3}ac+\frac{15}{4}b^2+25bc$ by $-\frac{2}{3}a+5b$.
32. Divide $-2x^5y^{-8}+17x^6y^{-4}-5x^7-24x^8y^4$ by $-x^2y^{-5}+7x^3y^{-1}+8x^4y^3$.
33. Divide $x^{pq}-1$ by x^p-1, and write down the last three terms of the quotient when q denotes an integer.
34. Divide $a^nb-ab^n-a^nc+ac^n+b^nc-bc^n$ by $(a-b)(a-c)$.

GREATEST COMMON MEASURE.

35. Find the greatest common measure of $a^2-ab-12b^2$ and $a^2+5ab+6b^2$.
36. Find the greatest common measure of $21x^3-26x^2+8x$ and $6x^2-x-2$.
37. Find the greatest common measure of $5x^3+2x^2-15x-6$ and $-7x^3+4x^2+21x-12$.
38. Find the greatest common measure of $x^4+x^3y-9x^2y^2+11xy^3-4y^4$ and $x^4-x^3y-3x^2y^2+5xy^3-2y^4$.
39. Find the greatest common measure of $3x^2+(4a-2b)x-2ab+a^2$ and $x^3+(2a-b)x^2-(2ab-a^2)x-a^2b$.
40. Find the greatest common measure of $6a^4x^3-10a^2x^4y-9a^3x^2y^2+15ax^3y^3$ and $10a^4xy^2-15a^3y^4+8a^2x^2y^3-12axy^5$.
41. Find the greatest common measure of $ab+2a^2-3b^2+4bc+ac-c^2$ and $2a^2-9ac-5ab+4c^2-8bc-12b^2$.
42. Find the greatest common measure of $e^{2x}a^3+e^{2x}-a^3-1$ and $e^{2x}a^2+2e^xa^2-2e^x+a^2-1$.
43. Find the greatest common measure of $3x^3-7x^2y+5xy^2-y^3$, $x^2y+3xy^2-3x^3-y^3$, and $3x^3+5x^2y+xy^2-y^3$.

LEAST COMMON MULTIPLE.

44. Find the least common multiple of $6a^2$, $9ax^3$, and $24x^5$.

45. Find the least common multiple of a^3+x^3, and a^2-x^2.

46. Find the least common multiple of

$$x^2-1,\ x^2+2x-3,\ \text{and}\ x^3-7x^2+6x.$$

47. Find the least common multiple of

$$21x^2-26x+8,\ \text{and}\ 7x^3-4x^2-21x+12.$$

48. Find the least common multiple of

$$2x^3+(2a-3b)x^2-(2b+3ab)x+3b^2,\ \text{and}\ 2x^2-(3b-2c)x-3bc.$$

49. Find the least common multiple of

$$x^2-1,\ x^2+1,\ (x-1)^2,\ (x+1)^2,\ x^3-1,\ \text{and}\ x^3+1.$$

FRACTIONS.

50. Simplify the expression $\dfrac{1}{2x+y}+\dfrac{1}{2x-y}-\dfrac{3x}{4x^2-y^2}$.

51. Find the value of $\dfrac{x+y}{y}-\dfrac{x}{x+y}-\dfrac{x^3-x^2y}{x^2y-y^3}$.

52. Reduce $\dfrac{a+\dfrac{b-a}{1+ba}}{1-a\dfrac{b-a}{1+ba}}\times\dfrac{a}{b}$.

53. Reduce $\dfrac{a}{b}-\dfrac{(a^2-b^2)x}{b^2}+\dfrac{a(a^2-b^2)x^2}{b^2(b+ax)}$.

54. Find the value of $\dfrac{3x-4y}{7}-\dfrac{2x-y-z}{3}+\dfrac{15x-4z}{12}$.

55. Find the value of $1+x+x^2+\dfrac{x^3}{1-x}$.

56. Find the value of $\dfrac{1}{4(1+x)}+\dfrac{1}{4(1-x)}+\dfrac{1}{2(1+x^2)}$.

57. Find the value of $\dfrac{3}{4(1-x)^2}+\dfrac{3}{8(1-x)}+\dfrac{1}{8(1+x)}-\dfrac{1-x}{4(1+x^2)}$.

58. Find the value of $\dfrac{a+c}{(a-b)(x-a)}-\dfrac{b+c}{(a-b)(x-b)}$.

59. Find the value of $\dfrac{1}{(x+1)(x+2)}-\dfrac{1}{(x+1)(x+2)(x+3)}$.

60. Find the value of $\dfrac{1}{(x+1)(x+2)}-\dfrac{3}{(x+1)(x+2)(x+3)}$.

61. Find the value of

$$\frac{1}{(a-b)(a-c)(x-a)}+\frac{1}{(b-a)(b-c)(x-b)}+\frac{1}{(c-a)(c-b)(x-c)}.$$

62. Find the value of

$$\frac{a}{(a-b)(a-c)(x-a)}+\frac{b}{(b-a)(b-c)(x-b)}+\frac{c}{(c-a)(c-b)(x-c)}.$$

63. Find the value of

$$\frac{a^2}{(a-b)(a-c)(x-a)}+\frac{b^2}{(b-a)(b-c)(x-b)}+\frac{c^2}{(c-a)(c-b)(x-c)}.$$

64. Find the value of

$$\frac{a^2+ha+k}{(a-b)(a-c)(x-a)}+\frac{b^2+hb+k}{(b-a)(b-c)(x-b)}+\frac{c^2+hc+k}{(c-a)(c-b)(x-c)}.$$

65. Find the value of $\frac{x^{3n}}{x^n-1}-\frac{x^{2n}}{x^n+1}-\frac{1}{x^n-1}+\frac{1}{x^n+1}$.

66. Find the value of

$$\frac{1+2x}{(3-x)(1+x)}+\frac{7}{(2+x)(1-3x)}+\frac{x}{(1+x)(2+x)}.$$

67. Find the value of

$$\frac{1-x}{1+x}+\frac{(1-x)(1-x^2)}{(1+x)(1+x^2)}+\frac{(1-x)(1-x^2)(1-x^3)}{(1+x)(1+x^2)(1+x^3)}.$$

68. Find the value of $\frac{1-x}{1+x}+\frac{1-x-x^2}{1+x+x^2}+\frac{1-x-x^2-x^3}{1+x+x^2+x^3}$.

69. Multiply $\frac{3x}{4}$ by $\frac{4x}{3y}$.

70. Multiply $8\frac{a^2b}{c}$ by $\frac{c^2d}{8a^3}$.

71. Multiply $a-\frac{x^2}{a}$ by $\frac{a}{x}+\frac{x}{a}$.

72. Multiply $\frac{a+2x}{a-x}$ by $\frac{(a-x)^2}{(a+2x)^2}$.

73. Multiply $\frac{x-y+z}{z}$ by $\frac{xz-z^2}{y}$.

74. Multiply $a-\frac{x^2}{a-x}$ by $x-\frac{x^2}{a+x}$.

75. Multiply $\frac{a^2+ax+x^2}{a^3-a^2x+ax^2-x^3}$ by $\frac{a^2-ax+x^2}{a+x}$.

76. Multiply $\frac{a^2}{x^2}-\frac{ab}{2xy}+\frac{b^2}{y^2}$ by $\frac{3a^2}{x^2}-\frac{2ab}{5xy}+\frac{b^2}{y^2}$.

77. Multiply $\frac{a^2}{b^3}+\frac{7c^2}{2a^4b^3}+\frac{2c^3d^4}{b^5}$ by $\frac{a^2}{b^3}-\frac{2c^3d^4}{b^5}-\frac{7c^2}{2a^4b^3}$.

78. Divide $\frac{5a^2}{b}$ by $\frac{3c^2}{5d}$.

79. Divide $\frac{a}{b}$ by $\frac{a-x}{c}$.

80. Divide $4x-\frac{3y^2}{x}$ by $9x+\frac{8y^3}{x^2}$. 81. Divide $\frac{a}{b}+\frac{c}{d}$ by $\frac{e}{f}-\frac{g}{h}$.

82. Divide $\frac{a}{b}-\frac{c}{d}+\frac{e}{f}$ by $\frac{s}{t}+\frac{x}{y}+\frac{z}{w}$.

83. Divide $\frac{1}{1+x}+\frac{x}{1-x}$ by $\frac{1}{1-x}-\frac{x}{1+x}$.

84. Divide $\frac{x+y}{x-y}+\frac{x-y}{x+y}$ by $\frac{x+y}{x-y}-\frac{x-y}{x+y}$.

85. Divide $\frac{x}{1+x}+\frac{1-x}{x}$ by $\frac{x}{1+x}-\frac{1-x}{x}$.

86. Divide $\frac{a^2}{bc}-\frac{2a}{d}+\frac{ac}{be}+\frac{bc}{d^2}-\frac{c^2}{de}$ by $\frac{a}{b}-\frac{c}{d}$.

87. Divide $\frac{a^2x^3}{bd}+\frac{abx^2}{c^2d}-\frac{acx^2}{d^2}-\frac{b^2x}{cd^2}+\frac{a^2x}{bc}-\frac{a}{b}$ by $\frac{ax}{c}-\frac{b}{d}$.

---

88. Reduce $\frac{x^2-4x+3}{x^2-2x-3}$ to its lowest terms.

89. Reduce $\frac{14a^2-7ab}{10ax-5bx}$ to its lowest terms.

90. Reduce $\frac{11x^2+11ax}{x^2-a^2}$ to its lowest terms.

91. Reduce $\frac{ac+by+ay+bc}{af+2bx+2ax+bf}$ to its lowest terms.

92. Reduce $\frac{x^3-39x+70}{x^2-3x-70}$ to its lowest terms.

93. Reduce $\frac{abc+9bc-5c^2}{12adf+18bdf-10cdf}$ to its lowest terms.

94. Reduce $\frac{acx^2+(ad-bc)x-bd}{a^2x^2-b^2}$ to its lowest terms.

95. Reduce $\frac{15x^3+35x^2+3x+7}{27x^4+63x^3-12x^2-28x}$ to its lowest terms.

96. Reduce to its lowest terms the fraction

$$\frac{4a^3cx-4a^3dx+24a^2bcx-24a^2bdx+36ab^2cx-36ab^2dx}{7abcx^3-7abdx^3+7ac^2x^3-7acdx^3+21b^2cx^3+21bc^2x^3-21b^2dx^3-21bcdx^3}.$$

97. Multiply $\frac{x^2-9x+20}{x^2-6x}$ by $\frac{x^2-13x+42}{x^2-5x}$, and reduce the product to its lowest terms.

98. Multiply $\frac{x^2+3x+2}{x^2+2x+1}$ by $\frac{x^2+5x+4}{x^2+7x+12}$, and reduce the product to its lowest terms.

99. Multiply $\frac{pr+(pq+qr)x+q^2x^2}{p-qx}$ by $\frac{ps+(pt-qs)x-qtx^2}{p+qx}$, and reduce the product to its lowest terms.

100. Reduce to its simplest form the quotient of

$$\frac{a^2+(2ac-b^2)x^2+c^2x^4}{a^2+2abx+(2ac+b^2)x^2+2bcx^3+c^2x^4} \div \frac{a^2+(ac-b)x^2+bcx^3}{a^2+(ac-b^2)x^2-bcx^3}.$$

101. Find the value of $\left(\frac{x}{a}-\frac{y}{b}\right)\frac{z}{c}+\left(\frac{x}{a}-\frac{z}{c}\right)\frac{y}{b}+\left(\frac{y}{b}-\frac{z}{c}\right)\frac{x}{a}$.

102. Find the value of $\dfrac{1}{\dfrac{1}{x+3}+\dfrac{1}{x-5}+\dfrac{1}{x+7}}$.

103. Find the value of $\dfrac{x}{1+\dfrac{x}{1+x+\dfrac{x}{1+x+x^2}}}$.

104. Find the value of

$$\frac{1}{1+x^{m-n}+x^{m-p}}+\frac{1}{1+x^{n-m}+x^{n-p}}+\frac{1}{1+x^{p-m}+x^{p-n}}.$$

105. Find the value of $\frac{x^3-1}{x^3-4x^2+4x-1}$ when $x=1$.

106. Find the value of $\frac{x^2+2x-35}{x^2-6x+5}$ when $x=5$.

107. Find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ when $x=\frac{4ab}{a+b}$.

108. Find the value of $\frac{xe^{2x}+1-e^{2x}-x}{e^{2x}-1}$ when $x=0$.

109. If $x^2=a^2+b^2$, and $y^2=c^2+d^2$, which is greater, xy or $ac+bd$?

110. Which is greater, n^3+1 or $n+n^2$?

111. Show that abc is greater than $(a+b-c)(a+c-b)(b+c-a)$ unless $a=b=c$.

112. Which is greater, $2(1+a^2+a^4)$ or $3(a+a^3)$?

113. Show that x^2-ax+b^2 is always positive, whatever be the value of x, if $4b^2-a^2$ is positive.

114. Find the cube of $1-2x+3x^2$.

115. Prove that $(a+b+c)^3+(a-b+c)^3+(a+b-c)^3+(b+c-a)^3$ $=2(a^3+b^3+c^3)+6(a^2b+ab^2+a^2c+b^2c+bc^2)-12abc$.

116. Find the value of $(a+c-b)^3+(a+b-c)^3+(b+c-a)^3$ $+24abc$.

117. Required the cube of e^x-e^{-x}.

118. Find the fifth power of $px\mp qz$.

119. Extract the square root of

$$4x^4-12x^3+25x^2-24x+16.$$

120. Extract the square root of
$$16b^4-16ab^2x+16b^2x^2+4a^2x^2-8ax^3+4x^4.$$

121. Extract the square root of
$$9x^2-30ax-3a^2x+25a^2+5a^3+\frac{a^4}{4}.$$

122. Extract the square root of
$$x^2+\frac{a^2}{9}+\frac{b^2}{4}+\frac{2ax}{3}-\frac{ab}{3}-bx.$$

123. Extract the square root of
$$9x^2-6xy+30xz+6xt+y^2-10yz-2yt+25z^2+10zt+t^2.$$

124. Extract the square root of
$$25\tfrac{3}{7}-\frac{20x}{7y}+\frac{9y^2}{16x^2}-\frac{15y}{2x}+\frac{4x^2}{49y^2}.$$

125. Extract the square root of
$$\frac{x^2}{y^2}\left(\frac{x^2}{4y^2}+1\right)+\frac{4y^2}{x^2}\left(\frac{y^2}{x^2}+1\right)+3.$$

126. Extract the square root of
$$\frac{x^2}{y^2}+\frac{y^2}{x^2}-\frac{x}{y}+\frac{y}{x}-1\tfrac{3}{4}.$$

127. Extract the cube root of
$$\frac{x^3}{8}+\frac{8}{27x^6}+\frac{2}{3x^3}+\frac{1}{2}.$$

128. Extract the cube root of
$$x^3+\frac{8}{x^3}-12x^2-\frac{48}{x^2}+54x+\frac{108}{x}-112.$$

129. Extract the cube root of
$$(a+1)^{6n}x^3-6ca^p(a+1)^{4n}.x^2+12c^2a^{2p}(a+1)^{2n}.x-8c^3a^{3p}.$$

130. Find the relations between p, q, r, s, when $x^4+px^3+qx^2+rx+s$ is a complete square.

SURDS.

131. Simplify $\sqrt{16a^2x}\pm\sqrt{4a^2x}$.

132. Simplify $a\sqrt{3a^2b}+\sqrt{12a^4b}$.

133. Simplify $b\sqrt[3]{8a^6b}+4a\sqrt[3]{a^3b^4}-\sqrt[3]{125a^6b^4}$.

134. Simplify $\sqrt[3]{\sqrt[4]{8a^3}}$ and $\sqrt[2]{\sqrt[5]{49a^4}}$.

135. Simplify $\sqrt{128}-2\sqrt{50}+\sqrt{72}-\sqrt{18}$.

136. Simplify $8\sqrt{\frac{3}{4}}+\frac{1}{2}\sqrt{12}-\frac{4}{3}\sqrt{27}-2\sqrt{\frac{3}{16}}$.

137. Simplify $3\sqrt[3]{\frac{1}{3}}-\sqrt[3]{72}$.

138. Simplify $4\sqrt{147}-3\sqrt{75}-6\sqrt{\frac{1}{3}}$.

139. Simplify $\sqrt[3]{40}+\sqrt[3]{135}$.

140. Simplify $\frac{3}{4}\sqrt[3]{8a^3b}-\frac{2b}{3}\sqrt[3]{\frac{a^6}{27b^5}}$.

141. Find the value of $\frac{1}{3}\sqrt[3]{a^2}\times 6\sqrt{a^3}$.

142. Simplify $a^{\frac{2}{3}}\times a^{\frac{7}{9}}\div a^3$.

143. Find the product of $1-\frac{a^{\frac{4}{3}}}{x^{\frac{3}{4}}}+a^{\frac{1}{3}}x^{\frac{1}{4}}-\frac{x^{\frac{5}{4}}}{a^{\frac{2}{3}}}$ by $a^{\frac{3}{4}}x^{-\frac{1}{2}}$.

144. Simplify $\sqrt{\frac{ay}{x}}.\sqrt[3]{\frac{bx}{y^2}}.\sqrt[6]{\frac{y^2}{b^2a^3}}$.

145. Multiply $x+2y^{\frac{1}{2}}+3z^{\frac{1}{3}}$ by $x-2y^{\frac{1}{2}}+3z^{\frac{1}{3}}$.

146. Multiply $a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}$ by $a^{\frac{1}{3}}-b^{\frac{1}{3}}$.

147. Multiply $a^{\frac{5}{2}}+a^2b^{\frac{1}{3}}+a^{\frac{3}{2}}b^{\frac{2}{3}}+ab+a^{\frac{1}{2}}b^{\frac{4}{3}}+b^{\frac{5}{3}}$ by $a^{\frac{1}{2}}-b^{\frac{1}{3}}$.

148. Multiply $\sqrt[3]{x^4}-2\sqrt[3]{(xy)^2}+\sqrt[3]{y^4}$ by $\sqrt[3]{x^2}-\sqrt[3]{y^2}$.

149. Multiply $\sqrt[3]{xy^2}+\sqrt[4]{xy^3}+\sqrt[5]{xy^4}$ by $\sqrt[3]{x^2y}-\sqrt[5]{x^4y}$.

150. Multiply $\frac{1}{2}\sqrt{a}+\frac{1}{3}\sqrt[3]{a}+\frac{1}{4}\sqrt[4]{a}$ by $\sqrt[4]{a}-\frac{3}{5}\sqrt[5]{a}$.

151. Multiply $\sqrt[3]{a^{-\frac{1}{2}}}+\sqrt[3]{\sqrt{a^{\frac{1}{2}}b}}$ by $\sqrt[3]{a^{-\frac{1}{2}}}-\sqrt{\sqrt[3]{a^{\frac{1}{2}}b}}$.

152. Multiply $pax^{\frac{5}{3}}+(p-1)a^2x^{\frac{2}{3}}+(p-2).a^3x^{-\frac{1}{3}}$ by $\frac{\sqrt[3]{x^4}}{a}-\sqrt[3]{x}$.

153. Multiply $\sqrt{-a}$ by $\sqrt{-b}$, and $\sqrt[4]{-a}$ by $\sqrt[4]{-b}$; also $\sqrt[6]{-a}$ by $\sqrt[6]{-b}$.

154. Find the continued product of
$x-1+\sqrt{2}$, $x-1-\sqrt{2}$, $x+2+\sqrt{3}$, and $x+2-\sqrt{3}$.

155. Find the continued product of
$4+2\sqrt{2}$, $1-\sqrt{3}$, $4-2\sqrt{2}$, $\sqrt{2}+\sqrt{3}$, $1+\sqrt{3}$, and $\sqrt{2}-\sqrt{3}$.

156. Find the product $\sqrt[m]{a+b}\times\sqrt[n]{a+b}.\times\sqrt[m]{a-b}\times\sqrt[n]{a-b}$.

157. Find the product of $5+2\sqrt{-3}$ by $2-\sqrt{-3}$.

158. Find the continued product of

$x-a\sqrt{-1}$, $x+a\sqrt{-1}$, $x+\frac{a}{2}(\sqrt{3}-\sqrt{-1})$, $x+\frac{a}{2}(\sqrt{3}+\sqrt{-1})$,

$x-\frac{a}{2}(\sqrt{3}-\sqrt{-1})$, and $x-\frac{a}{2}(\sqrt{3}+\sqrt{-1})$.

159. Raise $-2x^{\frac{1}{2}}y^{\frac{1}{3}}z^{\frac{1}{4}}$ to the 3rd, 4th and 6th powers.

160. Raise $-3x^2y^{\frac{2}{5}}z^3$ to the fifth power.

161. Raise $\sqrt[6]{(a^3b\sqrt[5]{a^3bc})}$ to the 7th power.

162. Divide $(a^{\frac{3}{5}})^7$ by $a^7 \times a^{\frac{3}{5}}$.

163. Find the cube of $(x+\sqrt{-y^2})^{\frac{1}{3}}-(x-\sqrt{-y^2})^{\frac{1}{3}}$.

164. Raise $\sqrt[3]{a}+\sqrt[3]{b}-\sqrt[3]{c}$ to the 3rd power.

165. Divide $\sqrt{\frac{4}{3}\sqrt{a^3}}-\sqrt{\frac{3}{4}\sqrt{a^3}}$ by $\frac{3\sqrt{3}}{\sqrt{a}}$.

166. Divide $x^{\frac{5}{2}}-4x^{\frac{3}{2}}-2x^{\frac{1}{2}}-x^2+6x$ by $x^{\frac{3}{2}}-4x^{\frac{1}{2}}+2$.

167. Divide $a^{\frac{5}{2}}+a^2b^{\frac{1}{3}}+a^{\frac{3}{2}}b^{\frac{2}{3}}+ab+a^{\frac{1}{2}}b^{\frac{4}{3}}+b^{\frac{5}{3}}$ by $a^{\frac{1}{2}}+b^{\frac{1}{3}}$.

168. Divide $16x-\frac{y^4}{16}$ by $2x^{\frac{1}{4}}-\frac{y}{2}$.

169. Divide $a-b^2$ by $a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b+b^{\frac{3}{2}}$.

170. Divide a^2-b by $\sqrt[3]{a}-\sqrt[6]{b}$.

171. Divide a^3-b^3 by $\sqrt[4]{a^3}+\sqrt[4]{b^3}$.

172. Divide $x^2-(4y)^2$ by $\sqrt{x}-2\sqrt{y}$.

173. Divide $x^{\frac{3}{2}}-\frac{1}{2}x^{\frac{1}{2}}$ by $x^2+\frac{1}{2}$ to 4 terms.

174. Divide $\sqrt[4]{x^3}$ by $\sqrt{x}-\sqrt[3]{y}$ to 4 terms.

175. Divide $\sqrt[12]{p^{10}q^9}-r\sqrt[10]{p^7}.\sqrt[6]{q^5}-\frac{3}{2}p\sqrt[4]{q^3}+\frac{3pqr}{2}\sqrt[30]{p^{-4}q^{-5}}$ by

$\sqrt{pq}-\frac{3}{2}\sqrt[6]{p^4q^3}$.

176. Divide $x-a$ by $x^{\frac{1}{n}}-a^{\frac{1}{n}}$.

177. Divide $2\sqrt[5]{100}$ by $3\sqrt[5]{80}$, and simplify the quotient.

178. Divide $\frac{5}{14}\sqrt[3]{\frac{2}{3}}$ by $\frac{5}{21}\sqrt[3]{\frac{9}{4}}$, and simplify.

179. Simplify $\sqrt[m]{x^3y^{mn}} \div \sqrt[n]{x^2y^{mn}}$.

180. Simplify $(\sqrt{a})^{\frac{2}{3}-\frac{1}{6}}+6\{a^3b.\sqrt[3]{a^3bc}\}^{\frac{1}{4}}$.

181. Simplify $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$.

182. Simplify $\sqrt[3]{\frac{1}{x}-x^2+3(x-1)}$.

183. Simplify $\frac{1}{x-1}+\frac{2}{2x+1-\sqrt{-3}}+\frac{2}{2x+1+\sqrt{-3}}$.

184. Simplify $\frac{x+\sqrt[3]{xy^2}-\sqrt[3]{x^2y}}{x+y}$.

185. Simplify $\left\{1+x-\frac{3}{2}\sqrt[3]{x}(1+\sqrt{x})+\sqrt{x}\left(2+\frac{9}{16}\sqrt[6]{x}\right)\right\}^{\frac{1}{2}}$.

186. Simplify $\frac{1}{4(1+\sqrt{x})}+\frac{1}{4(1-\sqrt{x})}+\frac{1}{2(1+x)}$.

187. Simplify $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}-\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$.

188. Simplify $\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}+\frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}$.

189. Simplify $\dfrac{\frac{1+\sqrt{5}}{2}x-2}{x^2-\frac{1+\sqrt{5}}{2}x+1}+\dfrac{\frac{1-\sqrt{5}}{2}x-2}{x^2-\frac{1-\sqrt{5}}{2}x+1}$.

190. Simplify $\frac{ab}{b-c}\pm\sqrt{\frac{a^2b^2}{(b-c)^2}-\frac{a^2b}{b-c}}$.

191. Simplify $a\sqrt{\frac{a^3b}{3a^2-6ac+3c^2}}+\frac{bc\sqrt{ab}}{a-c}$.

192. Simplify $\sqrt{\left\{\left(\frac{a^{-2m}}{b^{-2n}}\right)^{\frac{p}{m}}\right\}^{\frac{q}{n}}}$.

193. Simplify $\{ab^2\sqrt{ab^3}.\sqrt[3]{ab^4}.\sqrt[4]{ab^5}\}^{\frac{1}{5}}$.

---

194. Extract the cube roots of $\sqrt{27a^3x}$ and of $\sqrt[3]{27a^3x}$.

195. Extract the mth root of $-2^{\frac{1}{m}}.a^m.b^{2m}.c^2$.

196. Extract the square root of $-2+a^{2\sqrt{2}}+a^{-2\sqrt{2}}$.

197. Extract the square root of $1+\frac{41a}{16}-\frac{3+3a}{2}\sqrt{a}+a^2$.

198. Extract the square root of

$$\frac{9a^3}{4}-5a^{\frac{5}{2}}b^{\frac{1}{2}}+\frac{179a^2b}{45}-\frac{4a^{\frac{3}{2}}b^{\frac{3}{2}}}{3}+\cdot16ab^2.$$

199. Extract the square root of

$$x^m+\frac{1}{4}\sqrt[\frac{np}{2}]{b^px^{2n}}-\sqrt[n]{b}\sqrt[2p]{x^{mp+4}}.$$

200. Extract the square root of

$$\sqrt[x]{a^{-\frac{2}{y}}}+\sqrt[r]{a^{\frac{1}{y}}b}-2\sqrt[2]{\sqrt[r]{b}}.\sqrt[2]{\sqrt[x]{\sqrt[y]{\sqrt[r]{a^{x-2r}}}}}.$$

Extract the square roots of the following quantities.

201. $7+2\sqrt{10}$
202. $2+\sqrt{3}$.
203. $7+4\sqrt{3}$.
204. $16+6\sqrt{7}$.
205. $8+2\sqrt{7}$.
206. $28-10\sqrt{3}$.
207. $8-2\sqrt{15}$.
208. $41-12\sqrt{5}$.
209. $37-20\sqrt{3}$.
210. $4-\sqrt{7}$.
211. $1-4\sqrt{-3}$.
212. $-3-2\sqrt{-2}$.
213. $2\sqrt{-3}-2$.
214. $4\sqrt{-5}-1$.

215. $13-20\sqrt{-3}$. 216. $24\sqrt{-1}-7$.
217. $21-\sqrt{-400}$. 218. $2\sqrt{-2}-1$.
219. $5-12\sqrt{-1}$. 220. $-3-4\sqrt{-1}$.
221. $2\sqrt{-1}$. 222. $-18\sqrt{-1}$.
223. $0\cdot03\pm0\cdot04\sqrt{-1}$. 224. $a^2+2x\sqrt{a^2-x^2}$.
225. $xy-2x\sqrt{xy-x^2}$. 226. $x+y+z+2\sqrt{xz+yz}$.
227. $1+\sqrt{1-m^2}$. 228. $2+2(1-x)\sqrt{1+2x-x^2}$.
229. $ax-2a\sqrt{ax-a^2}$. 230. $2a+2\sqrt{a^2-b^2}$.
231. $a+b+c+d-2\sqrt{ac+bc+ad+bd}$.
232. $4mn+2(m^2-n^2)\sqrt{-1}$. 233. $2n\sqrt{-1}$.
234. $3\sqrt{6}+2\sqrt{12}$. 235. $3\sqrt{3}+2\sqrt{6}$.
236. $\sqrt{18}-\sqrt{16}$. 237. $\frac{\sqrt{18}}{2}+2$.
238. $8\sqrt{3}-6\sqrt{5}$. 239. $\frac{8}{7}\sqrt{21}-2\sqrt{3}$.
240. $6+\sqrt{8}-\sqrt{12}-\sqrt{24}$. 241. $9+2\sqrt{3}+2\sqrt{5}+2\sqrt{15}$.
242. $12-8\sqrt{2}+6\sqrt{3}-4\sqrt{6}$.
243. Find the value of $\sqrt{4+3\sqrt{-20}}+\sqrt{4-3\sqrt{-20}}$.

Extract the cube roots of the following quantities.

244. $38+17\sqrt{5}$. 245. $10\sqrt{7}+22$.
246. $24\sqrt{21}-64$. 247. $7-5\sqrt{2}$.
248. $10-6\sqrt{3}$. 249. $11\sqrt{2}+9\sqrt{3}$.
250. $-11-2\sqrt{-1}$. 251. $-4-10\sqrt{-2}$.

Extract the fourth roots of the following quantities.

252. $97+28\sqrt{12}$. 253. $49+20\sqrt{6}$.
254. $-79-8\sqrt{-5}$. 255. $14+8\sqrt{3}$.
256. $\frac{17}{3}-4\sqrt{2}$. 257. $-16a^4$.
258. $-64a^8$.

Extract the fifth roots of the following quantities.

259. $29\sqrt{2}-41$. 260. $41+29\sqrt{2}$.

261. Extract the cube root of $\frac{1}{8}a^3-\frac{3}{2}a^2b^{\frac{1}{4}}+6ab-8b^{\frac{3}{2}}$.

262. Extract the cube root of

$$\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}-\frac{3x}{a}+\frac{6x^{\frac{1}{2}}}{a^{\frac{1}{2}}}-7+\frac{6a^{\frac{1}{2}}}{x^{\frac{1}{2}}}-\frac{3a}{x}+\frac{a^{\frac{3}{2}}}{x^{\frac{3}{2}}}.$$

263. Extract the cube root of

$$x^{\frac{3m}{n}}.y^{-\frac{3n}{m}}-3x^{\frac{2m^2-n^2}{mn}}.y^{\frac{m^2-2n^2}{mn}}+3x^{\frac{m^2-2n^2}{mn}}.y^{\frac{2m^2-n^2}{mn}}-x^{-\frac{3n}{m}}.y^{\frac{3m}{n}}.$$

264. Extract the fourth root of

$$x^4y^{-\frac{4}{3}}-4x^{\frac{5}{2}}.y^{-\frac{1}{3}}+6xy^{\frac{2}{3}}-4x^{-\frac{1}{2}}y^{\frac{5}{3}}+x^{-2}.y^{\frac{8}{3}}.$$

265. Extract the fourth root of

$$\frac{\sqrt[20]{x^3}}{16}-\frac{5x^5\sqrt[5]{y^4}}{2}+\frac{75\sqrt[20]{x^6}\sqrt[5]{y^8}}{2}-250\sqrt[3]{x^5}\sqrt[5]{y^{12}}+625\sqrt[5]{y^{16}}.$$

266. Reduce $\dfrac{\sqrt[3]{5{\cdot}12}+\sqrt[3]{0{\cdot}03375}}{\sqrt[3]{80}-\sqrt[3]{0{\cdot}01}}$ to its equivalent simple decimal.

Reduce the following fractions to equivalent fractions with rational denominators.

267. $\dfrac{1+\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}$.

268. $\dfrac{1}{2\sqrt{2}-\sqrt{3}}$.

269. $\dfrac{3}{\sqrt{5}-\sqrt{2}}$.

270. $\dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.

271. $\dfrac{31}{4\sqrt{5}+3\sqrt{2}}$.

272. $\dfrac{1+\sqrt{2}}{1+\sqrt{2}+\sqrt{3}}$.

273. $\dfrac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$.

274. $\dfrac{1}{\sqrt{3}+\sqrt[3]{5}}$.

275. $\dfrac{2-\sqrt[3]{3}}{2+\sqrt[3]{3}}$.

276. $\dfrac{2+\sqrt[3]{3}}{2-\sqrt[3]{3}}$.

277. $\dfrac{1}{a^3-\sqrt[3]{b}}$.

278. $\dfrac{1}{\sqrt{3^3}-\sqrt[3]{5^2}}$.

279. $\dfrac{\sqrt[6]{9}-\sqrt{2}}{\sqrt[6]{9}+\sqrt{2}}$.

280. Find the value of $\dfrac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}}$ when $x=\dfrac{1}{2}\left\{\sqrt{\dfrac{a}{b}}-\sqrt{\dfrac{b}{a}}\right\}$.

281. Find the value of $\dfrac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$ when $x=\dfrac{2ab}{b^2+1}$.

282. Prove that $\sqrt{(a+b+c)^2-4bc}$ is always possible.

283. If $2p=\left(x+\dfrac{1}{x}\right)$ and $2q=\left(y+\dfrac{1}{y}\right)$, find the value of $pq-\sqrt{1-p^2}.\sqrt{1-q^2}$.

284. Which is greater, $\left(\dfrac{1}{2}\right)^{\frac{1}{2}}$ or $\left(\dfrac{2}{3}\right)^{\frac{2}{3}}$?

285. Which is greater, $\sqrt[5]{2}$ or $\sqrt[8]{3}$?

286. Which is greater, $\sqrt[3]{9}$ or $\sqrt[4]{18}$?

287. Which is greater, $\sqrt{2}+\sqrt{7}$ or $\sqrt{3}+\sqrt{5}$?

288. Which is greater, $\sqrt{10}+\sqrt{7}$ or $\sqrt{19}+\sqrt{3}$?

289. Which is greater, $\sqrt[3]{2}+\sqrt[3]{5}$ or 3?

290. If a be greater than 1, show which is greater, $a+\sqrt{a}$ or $1+\sqrt{a^3}$; $a^{\frac{3}{2}}-1$ or $(a-1)^{\frac{3}{2}}$.

291. Of the quantities $\sqrt{2}$, $\sqrt[3]{3}$, and $\sqrt[5]{5}$; determine the greatest and least.

292. If $(x+y)^{\frac{2}{3}}-(x-y)^{\frac{2}{3}}$ be greater or less than 1, show that $4xy-3(x^2-y^2)^{\frac{2}{3}}$ is greater or less than 1 respectively.

293. If m be greater than 3, prove that $\sqrt{m}>\sqrt[3]{m+1}$ and $\sqrt[3]{m}>\sqrt[4]{m+1}$.

294. Reduce $\sqrt[4]{-1}$ to the form $\alpha+\beta\sqrt{-1}$.

295. Express $\dfrac{a+b\sqrt{-1}}{c+d\sqrt{-1}}$ in the form $A+B\sqrt{-1}$.

296. Of the two expressions $(a+b\sqrt{-1})^{\frac{1}{4}}+(a-b\sqrt{-1})^{\frac{1}{4}}$ and $(a+b\sqrt{-1})^{\frac{1}{4}}-(a-b\sqrt{-1})^{\frac{1}{4}}$, show which is possible and which impossible.

297. Show how to find $\sqrt[3]{-1}$ in the form $\alpha\pm\beta\sqrt{-1}$.

298. If $(a^2+b^2)(c^2+d^2)=x^2+y^2$, show how to find x and y.

299. Show how to resolve the expression $a^2+b^2+c^2+d^2-2(ad+bc)$ into two simple factors.

300. Prove that
$$\frac{\{2xy-\sqrt{(1-x^4)(1-y^4)}\}^2}{\{2x^2+(1-x^2)(1+y^2)\}\{2y^2+(1+x^2)(1-y^2)\}}$$
is equal to
$$\left\{\frac{xy-\sqrt{\dfrac{(1-x^2)(1-y^2)}{(1+x^2)(1+y^2)}}}{1+xy\sqrt{\dfrac{(1-x^2)(1-y^2)}{(1+x^2)(1+y^2)}}}\right\}^2.$$

301. Given $2(x^2+y^2-x-y)+1=0$, find x and y.

EQUATIONS.

Solve the following equations.

302. $bx+2x-a=3x+2c$.

303. $\dfrac{x}{a}+\dfrac{x}{b}=c$.

304. $3x+\dfrac{5}{4}x=34$.

305. $5(x+1)-2=3(x-5)$.

306. $3(x-2)+4=4(3-x)$.

307. $5-3(4-x)+4(3-2x)=0$.

308. $3(x-3)-2(x-2)+x-1=x+3+2(x+2)+3(x+1)$.

309. $5(5x-6)-4(4x-5)+3(3x-2)-2x-16=0$.

310. $\dfrac{ax}{b}+\dfrac{cx}{f}+g=qx+\dfrac{1}{f}(fh-cx)$.

311. $\frac{3x}{4}+\frac{7x}{15}+\frac{11x}{6}-366=0.$

312. $\frac{x+1}{2}+\frac{x+2}{3}=\frac{5-x}{4}+14.$

313. $\frac{x}{2}-\frac{5x+4}{3}=\frac{4x-9}{3}.$

314. $x-\frac{x-2}{3}=\frac{x+23}{4}-\frac{10+x}{5}.$

315. $\frac{2x-5}{3}+x=\frac{3x-2}{5}+3.$

316. $\frac{x}{2}+\frac{x}{3}=x-7.$

317. $\frac{x}{2}-\frac{x}{3}=\frac{x}{4}-1.$

318. $\frac{x}{8}-\frac{x-1}{2\frac{1}{2}}=\frac{3x-4}{15}+\frac{x}{12}.$

319. $\frac{x}{2}-\frac{x}{3}+\frac{x}{4}=2-\frac{x}{6}+\frac{5x}{12}.$

320. $\frac{x}{6}-8\tfrac{3}{5}=2\left(\frac{3x}{5}-1\right)-\frac{x+8}{3}+1\tfrac{2}{3}.$

321. $\frac{9x+7}{2}-\left(x-\frac{x-2}{7}\right)=36.$

322. $\frac{3x^2-2x+1}{5}=\frac{(7x-2)(3x-6)}{35}+\frac{18}{20}.$

323. $1-\frac{x}{2}\left(1-\frac{3}{4x}\right)=\frac{2}{3}\left(3-\frac{5x}{2}\right)+5\tfrac{13}{40}.$

324. $\frac{a}{bx}-\frac{b}{ax}=a^2-b^2.$

325. $\frac{1}{x}+\frac{1}{2x}-\frac{1}{3x}=\frac{7}{3}.$

326. $\frac{x-a}{3}-\frac{2x-3b}{5}-\frac{a-x}{2}=10a+11b.$

327. $\frac{7x+5}{23}+\frac{9x-1}{10}-\frac{x-9}{5}+\frac{2x-3}{15}=23\tfrac{1}{3}.$

328. $\frac{2x+1}{29}-\frac{402-3x}{12}=9-\frac{471-6x}{2}.$

329. $\frac{11x-13}{25}+\frac{19x+3}{7}-\frac{5x-25\frac{1}{3}}{4}=28\tfrac{1}{7}-\frac{17x+4}{21}.$

330. $\frac{3x-\frac{2}{3}(1+x)}{4}+\frac{1-\frac{x}{5}}{5\frac{1}{2}}=\frac{2\frac{2}{5}+\frac{x-1}{25}}{2\frac{1}{5}}.$

331. $\frac{x-\frac{51}{26}}{2}-\frac{1-3x}{6\frac{1}{2}}=x-\frac{5x-\frac{1-\cdot 3x}{\cdot 4}}{39}.$

332. $\frac{a}{b+x}+\frac{a}{b-x}=c.$

333. $\frac{2(3-4x)}{3-x}+\frac{3}{1-x}=8.$

334. $\dfrac{x-7}{x+7}=\dfrac{2x-15}{2x-6}-\dfrac{1}{2(x+7)}.$

335. $\dfrac{7x+1}{x-1}=\dfrac{35}{9}\left(\dfrac{x+4}{x+2}\right)+\dfrac{28}{9}.$ 336. $\dfrac{66x+1}{1{\cdot}5x+1}+\dfrac{4x+5}{{\cdot}5x-1}=52.$

337. $\dfrac{6-5x}{15}-\dfrac{7-2x^2}{14(x-1)}=\dfrac{3x+1}{21}-\dfrac{2x-\dfrac{11}{5}}{6}+\dfrac{1}{105}.$

338. $\dfrac{x}{2}-\dfrac{\dfrac{2x-3}{3}-\dfrac{3x-1}{4}}{\dfrac{1}{2}(x-1)}=\dfrac{3}{2}.\dfrac{x^2+2}{3x-2}.$

339. $\sqrt{12+x}=2+\sqrt{x}.$ 340. $\sqrt{5}.\sqrt{x+2}=\sqrt{5x}+2.$

341. $(3\sqrt{x}+5)^2=115-(7-9x).$

342. $6x+a:4x+b=3x-b:2x-a.$

343. $\dfrac{3x-1}{\sqrt{3x}+1}=1+\dfrac{\sqrt{3x}-1}{2}.$

344. $\sqrt{x}-\sqrt{a+x}=\sqrt{\dfrac{a}{x}}.$

345. $\dfrac{1}{3}\sqrt[3]{17x-6}+\dfrac{3}{8}=1\tfrac{1}{24}.$ 346. $a+x\pm\sqrt{a^2+x^2}=b.$

347. $\sqrt{x}+\sqrt{a-\sqrt{ax+x^2}}=\sqrt{a}.$

348. $\sqrt{a-x}+2\sqrt{a+x}=\sqrt{a-x+\sqrt{ax+x^2}}.$

349. $\sqrt{1+x+x^2}=a-\sqrt{1-x+x^2}.$

350. $\dfrac{1}{a}\sqrt{a+x}+\dfrac{1}{x}\sqrt{a+x}=\dfrac{1}{b}\sqrt{x}.$

351. $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1.$

352. $\sqrt{1-x}+\sqrt{1-x+\sqrt{1+x}}=\sqrt{1+x}.$

353. $x+\sqrt{a^2+x^2}=\dfrac{na^2}{\sqrt{a^2+x^2}}.$

354. $\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}=a\sqrt{\dfrac{x}{x+\sqrt{x}}}.$

355. $\sqrt{a+\sqrt{a^2-x^2}}+\sqrt{a-\sqrt{a^2-x^2}}=n\sqrt{\dfrac{a+x}{a+\sqrt{a^2-x^2}}}.$

356. $\sqrt[m]{a+x}=\sqrt[2m]{x^2+8ax+b^2}.$

357. $\sqrt[3]{1+x}+\sqrt[3]{1-x}=\sqrt[3]{2}.$

358. $\sqrt{x}+2\sqrt{2a-x}=\sqrt{x+\sqrt{2a^2-3ax+x^2}}.$

359. $\frac{a+x}{\sqrt{a}+\sqrt{a+x}}+\frac{a-x}{\sqrt{a}+\sqrt{a-x}}=\sqrt{a}.$

360. $\frac{1+x-\sqrt{2x+x^2}}{1+x+\sqrt{2x+x^2}}=a\frac{\sqrt{2+x}+\sqrt{x}}{\sqrt{2+x}-\sqrt{x}}.$

361. $\sqrt{1-a}\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}+\sqrt{1+a}\left(\frac{1-x}{1+x}\right)^{\frac{1}{4}}=2\sqrt[4]{1-a^2}.$

362. $\frac{1+x^3}{(1+x)^2}+\frac{1-x^3}{(1-x)^2}=a.$

363. $\sqrt{(1+x)^2-ax}+\sqrt{(1-x)^2+ax}=x.$

364. $\sqrt{\frac{3a}{4}-x}+\sqrt{3ax-x}=\frac{3a}{2}\sqrt{1-4x}.$

365. $ax+1=\frac{2ax\sqrt{x+a^2}}{a+\sqrt{x+a^2}}.$

366. $\sqrt[4]{x^4-1}+x\sqrt{x^4-1}=x^3.$

367. $\frac{\sqrt{a}-\sqrt{a-\sqrt{a^2-ax}}}{\sqrt{a}+\sqrt{a-\sqrt{a^2-ax}}}=b.$

368. $\frac{\sqrt{1+x}-1}{\sqrt{1-x}+1}+\frac{\sqrt{1-x}+1}{\sqrt{1+x}-1}=a.$

369. $\frac{1-ax}{1+ax}\cdot\sqrt{\frac{1+bx}{1-bx}}=1.$

---

370. $ax=by$, and $x+y=c$.
371. $2x=11+9y$, and $3x-12y=15$.
372. $3x-7y=7$, and $11x+5y=87$.
373. $x+ay=b$, and $ax-by=c$.
374. $x-\frac{1}{7}(y-2)=5$, and $4y-\frac{1}{3}(x+10)=3$.
375. $y(3+x)=x(7+y)$, and $5x+9=2x+11$.
376. $\frac{x}{3}+\frac{y}{5}=8$, and $\frac{x}{9}-\frac{y}{10}=1$.
377. $\frac{x}{9}+\frac{y}{8}=43$, and $\frac{x}{8}+\frac{y}{9}=42$.
378. $\frac{m}{x}+\frac{n}{y}=a$, and $\frac{n}{x}+\frac{m}{y}=b$.
379. $\frac{1}{3x}+\frac{1}{5y}=\frac{2}{9}$, and $\frac{1}{5x}+\frac{1}{3y}=\frac{1}{4}$.
380. $\frac{4x+5y}{40}=x-y$, and $\frac{2x-y}{3}+2y=\frac{1}{2}$.
381. $2x+\cdot4y=1\cdot2$, and $3\cdot4x-\cdot02y=\cdot01$.

382. $\frac{1}{3}(x+y)+\frac{1}{4}(x-y)=59$, and $x:3y=11:5$.

383. $\frac{3x+4y+3}{10}-\frac{2x+7-y}{15}=5+\frac{y-8}{5}$,

and $\frac{9y+5x-8}{12}-\frac{x+y}{4}=\frac{7x+6}{11}$.

384. $2x-\frac{y+3}{4}=7+\frac{3y-2x}{5}$, and $4y+\frac{x-2}{3}=26\tfrac{1}{2}-\frac{2y+1}{2}$.

385. $2{\cdot}4x+0{\cdot}32y-\frac{0{\cdot}18x-0{\cdot}025}{0{\cdot}25}=0{\cdot}8x+\frac{5{\cdot}2+0{\cdot}01y}{{\cdot}5}$,

and $\frac{0{\cdot}2y+{\cdot}5}{1{\cdot}5}=\frac{0{\cdot}49x-0{\cdot}7}{4{\cdot}2}$.

386. $6x-\frac{2y-x}{23-x}=20-\frac{59-12x}{2}$, and $3y+\frac{y-3}{x-18}=30-\frac{73-9y}{3}$.

387. $\frac{4x-2y+3}{3}-\frac{18-x+5y}{7}=\frac{x}{4}-\frac{y}{5}-\frac{1}{7}-7\tfrac{7}{10}$,

and $2x-y+15:y-2x+15=\frac{x}{3}-\frac{y}{4}+\frac{3}{4}:\frac{y}{4}-\frac{x}{3}+\frac{1}{12}$.

388. $\frac{4x-8y+5}{2}=\frac{10x^2-12y^2-14xy+2x}{5x+3y+3}+2$,

and $\sqrt{6+x}:\sqrt{6-y}=3:2$.

389. $\frac{x}{a}+\frac{y}{b}=1-\frac{x}{c}$, and $\frac{y}{a}+\frac{x}{b}=1+\frac{y}{c}$.

390. $\sqrt{ax}+\sqrt{by}=\frac{1}{2}(x+y)=a+b$.

391. $a(x^2+y^2)-b(x^2-y^2)=2a$, and $(a^2-b^2)(x^2-y^2)=4ab$.

392. $\sqrt{y}-\sqrt{20-x}=\sqrt{y-x}$, and $\sqrt{y-x}+\sqrt{20-x}:\sqrt{y-x}=5:$

393. $(a^2-b^2)(5x+3y)=(4a-b)2ab$,

and $a^2y-\frac{ab^2c}{a+b}+(a+b+c)bx=b^2y+(a+2b)ab$.

394. $5x+3y=65$,
$2y-z=11$,
$3x+4z=57$.

395. $x+2y+3z=17$,
$2x-3y+z=0$,
$3x+y-5x=-15$.

396. $3x-7y+4z=1$,
$-5x+9y-z=22$,
$x-2y+z=0$.

397 $4(y-x)=5z-22$,
$3z+4x=6y+2$,
$z-3y=14-10x$.

398. $y+\frac{z}{3}=\frac{x}{5}+5$,
$\frac{x-1}{4}-\frac{y-2}{5}=\frac{z+3}{10}$,
$x-\frac{2y-5}{3}=1\tfrac{3}{4}-\frac{z}{12}$.

399. $\frac{xy}{x+y}=1$,
$\frac{xz}{x+z}=2$,
$\frac{yz}{y+z}=3$.

400. $\frac{1}{x}+\frac{1}{y}=a,$
$\frac{1}{x}+\frac{1}{z}=b,$
$\frac{1}{y}+\frac{1}{z}=c.$

401. $xy=3(x+y),$
$xz=8(x+z),$
$yz=\frac{9}{7}(y+z).$

402. $\frac{10}{x}-\frac{4}{5y}+\frac{1}{z}=21\cdot6,$
$\frac{1}{3x}+\frac{5}{9y}+\frac{2}{z}=10\cdot3,$
$\frac{4}{5x}-\frac{1}{2y}+\frac{4}{z}=16\cdot1.$

403. $\frac{xy+86}{3x+2y}=\frac{4x-y}{5},$
$\frac{xz+4\frac{1}{2}}{x+3z}=\frac{x-z}{2},$
$\frac{yz+4\frac{1}{7}}{2y+z}=\frac{y+3z}{7}.$

404. $\frac{2x+z-4}{12}+\frac{3y-6z+1}{13}=\frac{x-2}{4},$
$\frac{3x-2y+5}{5}-\frac{4x-5y+7z}{7}=\frac{2}{7}+\frac{3y-9z+6}{6},$
$\frac{x}{9}-y+3z=2.$

405. $x-y+z=0,$
$(a+b)x-(a+c)y+(b+c)z=0,$
$abx-acy+bcz=1.$

406. $5x-11\sqrt{y}+13\sqrt[3]{z}=22,$
$4x+6\sqrt{y}+5\sqrt[3]{z}=31,$
$x-\sqrt{y}+\sqrt[3]{z}=2.$

407. $xy^2z^3=108,$
$\frac{yz^2}{x}=18,$
$\frac{z}{yx^2}=\frac{3}{2}.$

408. $xyz=105,$
$\frac{x}{yz}=\frac{3}{35},$
$\frac{xy}{z}=\frac{15}{7}.$

409. $xyz=231,$
$xyw=420,$
$yzw=1540,$
$xzw=660.$

410. $9x-2z+u=41,$
$7y-5z-t=12,$
$4y-3x+2u=5,$
$3y-4u+3t=7,$
$7z-5u=11.$

---

411. $x^2-10x=24.$

412. $x^2-(a+b)x+ab=0.$

413. $\frac{x^2}{3}=9+\frac{x}{2}.$

414. $\frac{3x^2}{4}-(2x^2-3)=\frac{16x^2+9}{5}.$

415. $\frac{5x^2-7}{7x^2-5}+\frac{33+9x^2}{3x^2-11}=12.$

416. $x+\frac{x^2+3}{x^2-5}=\frac{12+5x^3}{5(x^2-5)}.$

417. $x+11342=x^2.$

418. $1\cdot5x-\frac{11}{x}=\frac{13x}{5}.$

419. $\left(5x+\frac{1}{2}\right)^2=751+5x.$ 420. $x^2-111x=3400.$

421. $11x^2-11\frac{1}{4}=9x.$ 422. $4x^2-4x=80.$

423. $x^2=5(x+89)+5555.$ 424. $6x^2+5x-4=0.$

425. $x(x-10)=(6\frac{2}{5}-x)10.$ 426. $17x^2+19x=1848.$

427. $3x^2-12x+1=6x-23.$ 428. $3(x-2)^2=18+(8x+1).$

429. $12x^2-x-1740=0.$ 430. $7x-\frac{21x^3-16}{3x^2+4}=5.$

431. $x-\frac{x^3-8}{x^2+5}=2.$ 432. $\frac{x+16}{5}+\frac{11}{x}=\frac{4x-7}{3}.$

433. $\frac{4}{x-3}-\frac{3}{x+5}=\frac{1}{18}.$ 434. $\frac{3x}{x+1}+\frac{2x-5}{3x-1}=5\frac{1}{4}.$

435. $\frac{7-2x}{9}+\frac{5x^2+7}{5x^2-7}=14+\frac{51-4x}{18}.$

436. $\frac{4x}{9}+\frac{x-5}{x+3}=\frac{4x+7}{19}.$

437. $\frac{x^2}{\sqrt{a}+\sqrt{b}}-(\sqrt{a}-\sqrt{b})x=\frac{1}{(ab^2)^{-\frac{1}{2}}+(a^2b)^{-\frac{1}{2}}}.$

438. $\frac{2x(a-x)}{3a-2x}=\frac{a}{4}.$ 439. $\frac{a+x}{\sqrt{a-x}}+\frac{a-x}{\sqrt{a+x}}=2\sqrt{a}.$

440. $\frac{7}{x^2+5x}+\frac{21}{3x^2-8x}=\frac{16}{x}.$

441. $x+5=\sqrt{x+5}+6.$ 442. $x^2=21+\sqrt{x^2-9}.$

443. $x^2-2x+6\sqrt{x^2-2x+5}=11.$

444. $x^2-x+5\sqrt{2x^2-5x+6}=\frac{3x+33}{2}.$

445. $\sqrt{x+60}+\sqrt{x^2+9}=\frac{2\sqrt{x^3+60x^2+9x+540}+89}{\sqrt{x+60}+\sqrt{x^2+9}}.$

446. $x+\sqrt{x^2-ax+b^2}=\frac{x^2}{a}+b.$

447. $\frac{\sqrt{x^2+x+6}}{3}=\frac{18-\left(\frac{4}{3}\sqrt{x^2+x+6}-2\right)}{\sqrt{x^2+x+6}}.$

448. $9x-3x^2+4\sqrt{x^2-3x+5}=11.$

449. $\{(x-2)^2-x\}^2-(x-2)^2=88-(x-2).$

450. $mqx^2-mnx+pqx-np=0.$

451. $\frac{x}{8}+\frac{1}{2x}=\sqrt{\frac{x}{3}+\frac{1}{4}-\frac{2}{3}}.$

452. $\sqrt{x+\sqrt{2x-1}}-\sqrt{x-\sqrt{2x-1}}=\frac{3}{5}\sqrt{\frac{10x}{x+\sqrt{2x-1}}}.$

453. $(x+4)+\sqrt{\frac{x+4}{x-4}}=\frac{12}{x-4}$.

454. $\frac{x^2}{4}=\frac{x-12}{x^2-18}$. 455. $\frac{x^2}{2x-5}=\frac{3}{x^2-17}$.

456. $\frac{3\sqrt{x}-\sqrt{x^3}}{x+2}=\frac{5+9\sqrt{x}-6x}{6\sqrt{x}-9}$.

457. $(\sqrt{a}+\sqrt{x})^{\frac{1}{3}}+(\sqrt{a}-\sqrt{x})^{\frac{1}{3}}=\sqrt[6]{a}$.

458. $\sqrt[4]{a+x}+\sqrt[4]{a-x}=h$.

459. $(a+x)^{\frac{2}{3}}+4(a-x)^{\frac{2}{3}}-5(a^2-x^2)^{\frac{1}{3}}=0$.

460. $(1+x)^{\frac{2}{5}}+(1-x)^{\frac{2}{5}}=(1-x^2)^{\frac{1}{5}}$.

461. $\sqrt[m]{(a^n+x^n)^2}+\sqrt[m]{(a^n-x^n)^2}=\frac{5}{2}\sqrt[m]{(a^{2n}-x^{2n})}$.

462. $x-3=\frac{3+4\sqrt{x}}{x}$.

463. $\frac{(27a+8x)^{\frac{2}{3}}}{15x^{\frac{13}{15}}}+\frac{8x^{\frac{2}{15}}}{3(27a+8x)^{\frac{1}{3}}}=\frac{8}{5\sqrt[5]{x}}$.

464. $\frac{\sqrt{a+bx^n}+\sqrt{a-bx^n}}{\sqrt{a+bx^n}-\sqrt{a-bx^n}}=c$.

465. $\frac{\sqrt{36x+1}+\sqrt{36x}}{\sqrt{36x+1}-\sqrt{36x}}=9$. 466. $\frac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}}=\frac{b}{x}$.

467. $\frac{ax+1+\sqrt{a^2x^2-1}}{ax+1-\sqrt{a^2x^2-1}}=\frac{b^2x}{2}$.

468. $\frac{x+\sqrt{x^2-a^2}}{x-\sqrt{x^2-a^2}}=\frac{x}{a}$. 469. $\frac{x^{(m-n)^2}+x^{-4mn}}{x^{(m-n)^2}-x^{-4mn}}=a^{\frac{r}{s}}$.

470. $\sqrt[2pq]{x^{p+q}}-\frac{1}{2}\cdot\frac{a^2-b^2}{a^2+b^2}(\sqrt[p]{x}+\sqrt[q]{x})=0$.

471. $\frac{16}{\sqrt{x^3}}+\frac{\sqrt{x}}{2}=\frac{6}{\sqrt{x}}$.

472. $3x^{\frac{3}{2}}-x^{-\frac{3}{2}}+2=0$. 473. $\frac{1}{x}+ax^{-\frac{1}{2}}=2a^2$.

474. $x^{\frac{7}{3}}+\frac{41\sqrt[3]{x}}{x}=\frac{97}{\sqrt[3]{x^2}}+\sqrt[6]{x^5}$.

475. $\sqrt{x}-\frac{8}{x}=\frac{7}{\sqrt{x}-2}$.

476. $(x+2)^2+2\sqrt{x}(x+2)-3\sqrt{x}=46+2x$.

477. $x^2+12x\sqrt{1+x}=27(1+x)$.

478. $x+7\sqrt[3]{x}=22$. 479. $x^3-px-p+1=0$.

480. $x^3-6x+9=0.$ 481. $x^3-6x^2+10x-8=0.$

482. $27x^3-135x^2+225x-117=0.$

483. $\frac{x}{a+x}+\frac{a}{\sqrt{a+x}}=\frac{b}{x}.$ 484. $2x^2+\sqrt{x^2+9}=x^4-9.$

485. $x^3-3x=2.$ 486. $x^3+px^2+px+1=0.$

487. $2x^3-x^2=1.$ 488. $2x\sqrt{1-x^4}=a(1+x^4).$

489. $\frac{(1+x)^2}{1+x^3}+\frac{(1-x)^2}{1-x^3}=a.$ 490. $\frac{1+x^3}{(1+x)^3}+\frac{1-x^3}{(1-x)^3}=c.$

491. $x^2-\frac{27x}{4}+25=7\sqrt{x(5-x)}.$

492. $x^4-2x^3+x=132.$

493. $x^4-8x^3+10x^2+24x+5=0.$

494. $x^4+\frac{13}{3}x^3-39x=81.$

495. $x^4+x^3-4x^2+x+1=0.$ 496. $x^4-x^3+\frac{5}{4}x^2-x+1=0.$

497. $x^6-7x^3=8.$

498. $\frac{1+x^3}{(1+x)^3}=h.$ 499. $\frac{1+x^4}{(1+x)^4}=\frac{1}{2},$ 499*. $\frac{1+x^5}{(1+x)^5}=n.$

---

500. $x-2y=2,$
$3xy=36.$

501. $x^2+y^2=41,$
$x-y=1.$

502. $\frac{2x+3y}{5}+\frac{7x-8y}{9}=3\frac{8}{9},$
$4x^2-5y^2=155.$

503. $x+4y=7,$
$x^2+y^2=10.$

504. $x+y=1,$
$x^2-xy=153.$

505. $6x-8y=3,$
$16x^2+xy=203\frac{7}{8}.$

506. $x^2-xy-y^2=\frac{1}{15}xy,$
$x-y=2.$

507. $7x^2-8xy=159,$
$5x+2y=7.$

508. $\frac{x^2}{y^2}=\frac{85-36x}{9y},$
$x-y=2.$

509. $14xy=\left(2x-\frac{3}{7}\right)\left(7y+\frac{1}{2}\right),$
$x^2y^2=(x^2-9)(y^2+1).$

510. $x^2+y^2=85,$
$xy=42.$

511. $\frac{y}{2x}=\frac{3}{4},$
$5xy+3x-y=492.$

512. $\sqrt{x}-\sqrt{y}=2,$
$x+y=20.$

513. $\sqrt[3]{x}+\sqrt[3]{y}=5,$
$x+y=35.$

514. $x^3-y^3=63,$
$x-y=3.$

515. $\sqrt{x}+\sqrt{y}=4,$
$\sqrt{x^3}+\sqrt{y^3}=28.$

516. $x^3+y^3=152,$
$x^2y+xy^2=120.$

517. $\sqrt[4]{x}+\sqrt[5]{y}=6,$
$\sqrt[4]{x^3}+\sqrt[5]{y^3}=126.$

518. $x+y=5,$
$x^4+y^4=257.$

519. $x-y=8,$
$x^4-y^4=14560.$

520. $x^5+y^5=a^5,$
$x+y=b.$

521. $x-y=1,$
$(x^2+y^2)(x^3-y^3)=247.$

522. $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{61}{\sqrt{xy}}+1,$
$\sqrt[4]{x^3y}+\sqrt[4]{xy^3}=78.$

523. $\sqrt{6\sqrt{x}+6\sqrt{y}}+\frac{1}{2}\sqrt{x}=9-\frac{1}{2}\sqrt{y},$
$x-y=12.$

524. $x^4+y^2=49+2x^2y,$
$x^4+y^4-x^2=20+(2x^2-1)y^2.$

525. $x+y+xy=11,$
$x^2y+xy^2=30.$

526. $5xy=84-x^2y^2,$
$x-y=6.$

527. $x^4+y^4=641,$
$xy(x^2+y^2)=290.$

528. $\frac{x^2+y^2}{10}=\frac{x+y}{3},$
$xy=8.$

529. $x^2+xy+y^2=52,$
$xy-x^2=8.$

530. $x^2+xy=a^2,$
$y^2-xy=b^2.$

531. $\sqrt{x^3}+\sqrt[3]{y^2}=3x,$
$\sqrt{x}+\sqrt[3]{y}=x.$

532. $x+3\sqrt{x+y}=18-y,$
$x^2-y^2=9.$

533. $\frac{1}{x}+\frac{1}{y}=\frac{1}{a},$
$\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{b^2}.$

534. $\frac{x}{y}-\frac{y}{x}=\frac{x+y}{x^2+y^2},$
$\frac{x^2}{y^2}-\frac{y^2}{x^2}=\frac{x-y}{y^2}.$

535. $\frac{x^2}{y^2}+\frac{2x+y}{\sqrt{y}}=20-\frac{y^2+x}{y},$
$x-4y+8=0.$

536. $3x^2+4y^2=7xy,$
$\sqrt{x^3}-y\sqrt{x}=\frac{2}{9}y^2.$

537. $x+\sqrt{x^2-y^2}=\frac{8}{y}(\sqrt{x+y}+\sqrt{x-y}),$
$\sqrt{(x+y)^3}-\sqrt{(x-y)^3}=26.$

538. $xy+\sqrt{x^2y^2-y^4}=\sqrt{x+y}+\sqrt{x-y},$
$\sqrt[4]{x+y}+\sqrt[4]{x-y}=y.$

539. $x^2+xy+y^2=a^2,$
$x+\sqrt{xy}+y=b.$

540. $x^3+y^3+x^2y+xy^2=13,$
$x^4y^2+x^2y^4=468.$

541. $\frac{y}{x}-\frac{81}{xy}=(2y+18)\frac{\sqrt{x}}{y},$
$y+3\sqrt{x^3}=9+3\sqrt{x^3y}.$

542. $x^4+y^4=1+2xy+3x^2y^2,$
$x^3+y^3=1+x+2y^2+2xy^2.$

543. $(x^4+y^4)^2+x^2y^2(x^2-y^2)^2+x^2-y^2=328,$
$x^2-y^2=3.$

544. $x^2+z^2=3xz,$
$x^5+z^5=2.$

545. $x^3y^2+xy^4=156,$
$2x^3y^2-x^2y^3=144.$

546. $x(bc-xy)=y(xy-ac),$
$xy(ay+bx-xy)$
$=abc(x+y-c).$

547. $x^2(b-y)=ay(y-n),$
$y^2(a-x)=bx(x-n).$

548. $$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}=\frac{9}{8y}(x+y),$$
$$(x^2+y)^2+x-y=2x(x^2+y)+506.$$

549. $$x^2+y^2=x+6,$$
$$x^3(2-x)+6y^2x-y^2(6x^2+y^2)=x-5.$$

550. $$5y+\frac{1}{5}\sqrt{x^2-15y-14}=\frac{1}{3}(x^2-108),$$
$$\frac{x^2}{8y}+\frac{y}{2}+\frac{2x}{3}=\sqrt{\frac{x^3}{3y}+\frac{x^2}{4}}.$$

551. $$(2+4xy-3x^2)^2=2-4x^2y^2+3x^4,$$
$$4y^4-(x^2+1)^2=(x^2-1)^2.$$

552. $$(x^2+y^2)(x+y)=15xy,$$
$$(x^4+y^4)(x^2+y^2)=85x^2y^2.$$

553. $$(x^2-xy+y^2)(x^2+y^2)=221,$$
$$(x^2-xy+y^2)(x^2+xy+y^2)=273.$$

554. $$\frac{\sqrt{y^2+1}+1}{y}=\frac{\sqrt{x+9}+3}{\sqrt{x}},$$
$$x(y+1)^2=36\left(y^3+\frac{16}{9}\right).$$

555. $$x^2+xy+y^2=37,$$
$$x^2+xz+z^2=28,$$
$$y^2+yz+z^2=19.$$

556. $$(\sqrt{x})^{\sqrt[4]{x}+\sqrt[4]{y}}=\sqrt[3]{y^4},$$
$$(\sqrt{y})^{\sqrt[4]{x}+\sqrt[4]{y}}=\sqrt[3]{x}.$$

557. $$\left(\frac{x}{a}\right)^\alpha\cdot\left(\frac{y}{b}\right)^\beta=a^{\alpha\beta},$$
$$\left(\frac{x}{b}\right)^\beta\cdot\left(\frac{y}{a}\right)^\alpha=b^{\alpha\beta}.$$

558. $xyz=9(x+y)$, $xyz=4(y+z)$, $xyz=16(x+z)$.

559. $$3\sqrt{3x^3-4y^3}=2x^{\frac{3}{4}}+64-\frac{201y^2}{4},$$
$$\frac{x^{\frac{3}{2}}+6y^2}{5y}=\frac{x^{\frac{3}{2}}+2y^2}{x^{\frac{3}{4}}+y}.$$

560. Form the equation whose roots are $+5$, -3.

561. Form the equation whose roots are 3, -2, and 7.

562. Form the equation whose roots are $+4\sqrt{3}$, $-4\sqrt{3}$, $5+2\sqrt{-1}$, and $5-2\sqrt{-1}$.

563. Form the equation whose roots are 0, ± 7, $\frac{1}{3}(1\pm\sqrt{-1})$.

564. If α, β represent the two values of x which satisfy the equation $ax^2+bx+c=0$, prove that $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{b^2}{ac}-2$. Exemplify this property in the equation $5x^2-9x+\frac{1}{4}=0$.

565. Eliminate x from the equations $a^2x^2+bx+c=0$, and $h^2x^2+kx+t=0$.

566. If $ax^3=by^3=cz^3$ and $x^{-1}+y^{-1}+z^{-1}=d^{-1}$, find the value of $ax^2+by^2+cz^2$ in terms of a, b, c and d.

567. Given $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{2x}-\sqrt{3y}}=a$, to find $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{2x}+\sqrt{3y}}$.

PROBLEMS IN SIMPLE EQUATIONS.

568. What number is that to which if 16 be added, four times this sum will be equal to ten times the number increased by 1?

569. What number is that which exceeds its seventh part by 12?

570. Find that number to which if its third part be added, the sum will equal 4 times the number diminished by 8.

571. Find a number such that if increased by 16 it will become 7 times as great as the third part of the original number.

572. Two coaches start at the same time from York and London, a distance of 200 miles. The one from London travels at $9\frac{1}{2}$ miles an hour, and that from York at $8\frac{3}{4}$. Where will they meet, and in what time from starting?

573. A and B begin to play with equal sums; A won £5, and then 3 times A's money was equal to 11 times B's: what had each at first?

574. A messenger starts on an errand at the rate of 4 miles an hour; another is sent an hour and a half after to overtake him: the latter walks at the rate of $4\frac{3}{4}$ miles an hour; when will he overtake the former?

575. A garrison of 500 men was victualled for 48 days; after 15 days it was reinforced and then the provisions were exhausted in 11 days: required the number of men in the reinforcement.

576. A person performs $\frac{2}{7}$ths of a piece of work in 13 days; he then receives the assistance of another person, and the two together finish it in 6 days; in what time could each do it separately?

577. A cistern is filled in 24 minutes by 3 pipes, one of which conveys 8 gallons more, and another 7 gallons less than the third, every three minutes. The cistern holds 1050 gallons. How much flows through each pipe in a minute?

578. A person buys four houses: for the second he gives half as much again as for the first; for the third, half as much again as for the second; and for the fourth, as much as for the first and third together: he pays £8000 for them all. What is the cost of each?

579. Find the fraction, which, if 1 be added to its numerator, becomes $\frac{1}{3}$; but if 1 be added to its denominator, becomes $\frac{1}{4}$.

580. Find the time in which A, B and C can together perform a piece of work, which requires 7, 6 and 9 days respectively when done singly.

581. A and B start to run a race to a certain post and back again. A returning meets B 90 yards from the post and arrives at the starting place 3 minutes before him. If he had returned immediately to meet B he would have met him at $\frac{1}{6}$ of the distance between the post and starting place. Find the length of the course and the duration of the race.

582. A waggon is furnished with a mechanical contrivance by which the difference of the number of revolutions of the fore- and hind-wheels is registered. The circumference of the fore-wheel is 7 feet, that of the hind-wheel is 9 feet; what is the distance gone over when the fore-wheel has made 500 revolutions more than the hind-wheel?

583. From a certain sum of money I took away a third part, and put in its stead £50; next from the sum thus augmented I took away a quarter, and put again in its stead £70; I then counted the money and found £120; what was the original sum?

584. Bacchus having caught Silenus asleep by the side of a full cask, seized the opportunity of drinking, which he continued for two-thirds of the time that Silenus would have taken to empty the whole cask. After that Silenus awakes and drinks what Bacchus had left. Had they both drank together it would have been emptied two hours sooner, and Bacchus would have drunk only half of what he left for Silenus: required the time in which each would have emptied the cask separately.

585. A bill of 25 guineas was paid with crowns and half-guineas, and twice the number of half-guineas exceeded three times that of the crowns by 17; how many were there of each?

586. A farm was rated at 3*s*. an acre, and the tenant on receiving back at his rent-day 10 per cent. of his rent, found that the sum returned amounted to £6 more than the whole rate. The next year the rates were increased 40 per cent., and he received back 12 per cent. of his rent, but the sum returned only just paid the whole rate. What was the rent of the farm, and of how many acres did it consist?

587. Find two numbers such, that if the first be added to four times the second, the sum is 29; and if the second be added to six times the first, the sum is 36.

588. A and B engaged in play; when A had lost £20 he had

only $\frac{1}{3}$rd the money which B had, but by continuing to play he not only won back his £20 but also £50 more with it, and then found he possessed half as much again as B: with what sum did they respectively begin?

589. A farmer mixes barley at 2*s.* 4*d.* a bushel with rye at 3*s.* a bushel, and wheat at 4*s.* a bushel, so that the whole is 100 bushels, and worth 3*s.* 4*d.* per bushel. Had he put twice as much rye, and 10 bushels more of wheat, the whole would have been worth exactly the same per bushel: how much of each kind was there?

590. A man and his wife could drink a barrel of beer in 15 days; after drinking together 6 days, the woman alone drank the remainder in 30 days: in what time would either alone drink it?

591. Find that number of two figures to which, if the number formed by changing the places of the digits be added, the sum is 121; and if the same two numbers be subtracted the remainder is 9.

592. A shopkeeper, on account of bad book-keeping, knows neither the weight nor the prime cost of a certain article which he had purchased; he knows however that if he had sold the whole at 30*s.* per lb. he would have gained £5 by it, and if he had sold it at 22*s.* per lb. he would have lost £15 by it: what was the weight and prime cost of the article?

593. To complete a certain work A requires m times as long a time as B and C together, B requires n times as long as A and C together, and C requires p times as long as A and B together; compare the times in which each would do it, and prove that

$$\frac{1}{m+1}+\frac{1}{n+1}+\frac{1}{p+1}=2.$$

PROBLEMS IN QUADRATIC EQUATIONS.

594. Find two numbers whose difference is to the greater as 2 to 9, and the difference of whose squares is 128.

595. Find three numbers in the proportion of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; the sum of whose squares is 724.

596. The sum of two numbers is 16; and the quotient of the greater divided by the less is to the quotient of the less by the greater as 25 is to 9: find them.

597. Required two numbers whose product is equal to the square of $\frac{2}{3}$ of one of them, and the difference of their squares exceeds by 1 the square of double the other.

598. A farmer bought some sheep for £72, and found that if he

had bought 6 more for the same money, he would have paid £1 less for each. How many sheep did he buy?

599. The sum of two numbers is 6, and the sum of their cubes is 72: find them.

600. A and B distribute £60 each among a certain number of persons: A relieves 40 persons more than B, and B gives to each person 5*s*. a-piece more than A. How many persons did they each relieve?

601. A detachment from an army was marching in regular column, with 5 men more in depth than in front; but upon the enemy coming in sight, the front was increased by 845 men; and by this movement the detachment was drawn up in 5 lines. Required the number of men.

602. A vintner draws a certain quantity of wine out of a full vessel that holds 256 gallons; and then filling the vessel with water, draws off the same quantity of liquor as before, and so on, for four draughts, when there were only 81 gallons of pure wine left. How much wine did he draw each time?

603. There is a number consisting of two digits, which, when divided by the sum of its digits, gives a quotient greater by 2 than the first digit. But if the digits be reversed, and then divided by a number greater by unity than the sum of the digits, the quotient is greater by two than the preceding quotient. Required the number.

604. What two numbers are those whose sum multiplied by the greater is 204; and whose difference multiplied by the less is 35?

605. There are two numbers such that the sum of the products of the first by 4 and of the second by 3 is 53; and the difference of their squares is 15. Required the two numbers.

606. Find that fraction whose terms consist of two consecutive numbers, and which, when added to twice its reciprocal, exceeds 2 by twice the reciprocal of the product of its terms.

607. The product of the sum and difference of the hypothenuse and a side of a right-angled triangle is equal to 2; and 4 times the sum of the squares of the hypothenuse and this side is equal to 5 times the sum of these two lines. Find the three sides of the triangle.

608. A and B travelled on the same road, and at the same rate, from N to M: at the 50th mile-stone from M, A overtook a flock of geese, which were proceeding at the rate of 3 miles in 2 hours: and 2 hours afterwards met a stage waggon, which was moving at the rate of $2\frac{1}{4}$ miles per hour. B overtook the same geese at the 45th mile-stone, and met the stage waggon exactly 40 minutes before he came to the 31st mile-stone. Where was B when A reached M?

609. Two boys set off in opposite directions from the right angle

of a triangular field, and ran along the sides without varying their velocities, which were in the ratio of 13 : 11. They met in the middle of the opposite side, and afterwards 30 yards from the point where they started. Required the lengths of the sides of the field.

610. A vintner sold 7 dozen of sherry and 12 dozen of claret for £50. He sold of sherry 3 dozen more for £10 than he did of claret for £6. Required the price of each.

611. There are three numbers, the difference of whose differences is 5; their sum is 44; and continued product 1950. What are the numbers?

612. A detachment of soldiers from a regiment being ordered to march on a particular service, each company furnished 4 times as many men as there were companies in the regiment; but these being found to be insufficient, each company furnished 3 more men; when their number was found to be increased in the ratio of 17 to 16. How many companies were there in the regiment?

613. A body of men are just sufficient to form a hollow equilateral wedge, three deep; and if 597 be taken away, the remainder will form a hollow square four deep, the front of which contains one man more than the square root of the number contained in a front of the wedge. What is the number of men?

614. There are two numbers such that 3 times the sum of their squares multiplied by the less is equal to 26 times the greater; and twice the difference of their squares multiplied by the greater is equal to 15 times the less. Required the two numbers.

615. The fore-wheel of a carriage makes 6 revolutions more than the hind-wheel in going 120 yards; but if the periphery of each wheel be increased one yard, it will make only 4 revolutions more than the hind-wheel in the same space. Required the circumference of each.

616. There are two sorts of metal, each being a mixture of gold and silver, but in different proportions. Two coins from these metals of the same weight are to each other in value as 11 to 17; but if to the same quantities of silver as before in each mixture double the former quantities of gold had been added, the values of two coins from them of equal weights would have been to each other as 7 to 11. Determine the proportion of gold to silver in each mixture; the values of equal weights of gold and silver being as 13 to 1.

RATIO, PROPORTION, AND VARIATION.

617. Prove that $a : b$ is a greater ratio than $ax : bx+h$; and a less ratio than $ax : bx-h$.

618. Of the ratios $a^4-b^4 : a^2-b^2$, and $a^5+b^5 : a^4-a^3b+a^2b^2-ab^3+b^4$, which is the greater?

619. Of the ratios $a^3+b^3 : a^2+b^2$, and $a^2+b^2 : a+b$, which is the greater?

620. If $a:b$ is a greater ratio than $c:d$, prove that $a+c:b+d$ is a ratio less than $a:b$, but greater than $c:d$.

621. If $a:b=c:d$, show that $7a+b:3a+5b=7c+d:3c+5d$.

622. If four quantities of the same kind be proportionals, show that the greatest and least together are greater than the other two together.

623. If $a:b=c:d$ and a be the greatest, prove that a^n+d^n is greater than b^n+c^n.

624. If $a:b=c:d$, prove that $(a-c)-(b-d)$ is equal to $(a-b)$ $(a-c)$ divided by a.

625. If $a:b=c:d$, show that $\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}+\frac{(a-b)(a-c)}{abc}$.

626. If $xy \propto x^2+y^2$, and 3, 4 be simultaneous values of x and y, express xy in terms of x^2 and y^2.

627. Given that $y^2 \propto x^2-a^2$; and when $x=\sqrt{a^2+b^2}$ then $y=\frac{b^2}{a}$; find the equation between x and y.

628. If $y^2 \propto x$ and a, $\pm 2a$ be simultaneous values of x and y respectively; find the equation between x and y.

629. If $x \propto y$, and $y \propto z$, prove that

$$(ax+by+cz) \propto (h\sqrt{xy}+k\sqrt{xz}+l\sqrt{yz}.)$$

630. What is the ratio of the radii of the inner and outer surfaces of a metallic shell whose weight is to that of a metallic ball of the same substance and volume as 7 : 8?

631. There are two vessels, A and B, each containing a mixture of water and wine, A in the ratio of 2 : 3, B in the ratio 3 : 7. What quantity must be taken from each, in order to form a third mixture which shall contain 5 gallons of water and 11 of wine?

632. The value of diamonds $\propto$ as the square of their weights, and the square of the value of rubies $\propto$ as the cube of their weight; a diamond of a carats is worth m times a ruby of b carats, and both together are worth £c; find the value of a diamond and ruby, each weighing x carats.

ARITHMETICAL PROGRESSION.

General Formulæ. $S=\{2a+(n-1)b\}\frac{n}{2}$, where S is the sum of n terms of an arithmetical series of which a is the first term and b the common difference. If h denote the nth term, then $h=a+(n-1)b$.

Sum the following series:

633. $1+3+5+7+\ldots$ to n terms and to 20 terms.

634. $2+7+12+17+\ldots$ to 101 terms.
635. $2+2\frac{1}{4}+2\frac{1}{2}+\ldots$ to 12 terms.
636. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\ldots$ to 7 terms.
637. $-5-1+3+\ldots$ to 12 terms.
638. $13+12\frac{1}{3}+11\frac{2}{3}+\ldots$ to n terms and to 40 terms.
639. $1+\frac{6}{7}+\frac{5}{7}+\ldots$ to 15 terms.
640. $6+\frac{11}{2}+5+\ldots$ to 25 terms.
641. $1+2\frac{2}{3}+4\frac{1}{3}+\ldots$ to 22 terms.
642. $2\frac{1}{3}+3\frac{5}{24}+4\frac{1}{12}+\ldots$ to 5 terms.
643. $\frac{5}{9}+\frac{2}{9}-\frac{1}{9}-\ldots$ to 101 terms.
644. $\frac{n-1}{n}+\frac{n-2}{n}+\frac{n-3}{n}+\ldots$ to n terms.
645. $(a+x)^2+(a^2+x^2)+(a-x)^2+\ldots$ to n terms.
646. $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+\ldots$ to n terms.

647. How many terms of the series 54, 51, 48... amount to 513?

648. How many terms of the series 19, 17, 15... amount to 91?

649. How many terms of the series 7, 9, 11... amount to 40?

650. How many terms of the series ·034, ·0344, ·0348... amount to 2·748?

651. The sum of an arithmetic series $s=1455$, the first term $a=5$, and the number of terms $n=30$; find (b) the common difference.

652. In an arithmetical progression it is observed that the fifth and ninth terms are 13 and 25; what is the seventh term?

653. The nth term of an arithmetical series is $\frac{1}{6}(3n-1)$; prove that the sum of n terms is $\frac{n}{12}(3n+1)$; and find the series.

654. Having given the first term $=\frac{10}{3}$, the common difference $=\frac{13}{9}$, and the sum of the series $=22$; find the number of terms.

655. In the series 1, 3, 5, &c., the sum of any even number of terms has to the sum of half that number of terms a constant ratio; find that ratio.

656. The sum of n terms of two arithmetical series are as $13-7n:1+3n$; find the ratio of their first terms, and also of their second terms.

657. Find the ratio of the latter half of $2n$ terms of any arithmetic series to the sum of $3n$ terms of the same series.

658. Determine the relation which must exist between a, b and c, that they may be respectively the pth, qth and rth terms of an arithmetical series.

659. If P and Q denote the pth and qth terms respectively of an arithmetic series; find the first term and the sum of n terms.

660. Given the first and last terms of an arithmetic series and the number of terms; determine the series.

661. Insert 9 arithmetic means between 1 and -1.

662. Insert 4 arithmetic means between 2 and -18.

663. Insert 3 arithmetic means between 117 and 477.

664. There are n arithmetic means between 1 and 31, such that the 7th: the $(n-1)$th$=5:9$; required n.

665. There are n arithmetic means between a and b, and between the pth and qth terms of these means there are r arithmetic means inserted; find the mth term of the last set.

666. The sum of m arithmetic means between 1 and 19 : sum of the first $m-2$ of them $=5:3$; find m.

667. The difference between the sums of m and n terms of an arithmetic series: the sum of $m+n$ terms $=m-n:m+n$.

668. If the $(p+q)$th term of an arithmetic series $=m$, and the $(p-q)$th$=n$; find the pth term and also the qth.

669. If s_1, s_2, $s_3 \dots s_r$ be the sums of r arithmetic series, each continued to n terms, the first terms being 1, 2, 3, ..., and the common differences 1, 3, 5 ... respectively; required the sum of $s_1+s_2+s_3+\dots+s_r$.

670. If s_n, s_{n+1}, $s_{n+2}\dots$ denote the sums of n, $n+1$, $n+2\dots$ terms of an arithmetic progression; required the sum of n terms of the series $s_n+s_{n+1}+\dots$.

671. In the two series 2, 5, 8 ... and 3, 7, 11 ... each continued to a 100 terms; find how many terms are identical.

672. A debt can be discharged in a year by paying 1*s.* the first week, 3*s.* the second, 5*s.* the third, and so on; required the amount of the last payment, and also of the whole debt.

673. There are six towns in the order of the letters A, B, C, D, E, F, whose distances from each other are in an increasing arithmetical progression; the distance from A to C is 16 miles, and from C to E is 24 miles; required their respective distances.

674. From two towns which were 168 miles distant from each other, two persons, A and B, set out to meet one another; A went 3 miles the first day, 5 the second, 7 the third, and so on; B went 4 miles the first day, 6 the second, 8 the third, and so on; in how many days after starting did they meet?

675. Five persons undertake to reap a field of 87 acres. The five terms of an arithmetical progression, whose sum is 20, will

express the times in which they can individually reap an acre, and together they can finish the undertaking in 60 days: in how many days can each separately reap an acre?

676. A person employed three workmen whose daily wages were in arithmetical progression. The number of days they worked was equal to the number of shillings that the second received per day. The whole amount of their wages was seven guineas, and the best workman received 28 shillings more than the worst: what were the daily wages of each workman?

GEOMETRICAL PROGRESSION.

General Formulæ. If a be the first term of a geometrical series, r the common ratio, and n the number of terms, then will the sum $s=\frac{a(r^n-1)}{r-1}$, and the nth term $z=ar^{n-1}$.

If Σ be the sum of a series convergent and infinitely extended, then $\Sigma=\frac{a}{1-r}$.

Sum the following series.

677. $1+3+9+\ldots$ to 9 terms.

678. $3+4\frac{1}{2}+6\frac{3}{4}+\ldots$ to 5 terms.

679. $\frac{1}{3}-\frac{1}{2}+\frac{3}{4}-\ldots$ to 8 terms.

680. $1-2+4-\ldots$ to n terms.

681. $\frac{2}{5}-\sqrt{\frac{2}{5}}+1-\ldots$ to 8 terms.

682. $\sqrt{\frac{3}{5}}-\sqrt{6}+2\sqrt{15}-\ldots$ to 8 terms.

683. $\frac{1}{5}-\frac{2}{15}+\frac{4}{45}-\ldots$ to n terms and to infinity.

684. $9-6+4-\ldots$ to 9 terms and to infinity.

685. $\frac{2}{3}-\frac{1}{2}+\frac{3}{8}-\ldots$ to infinity.

686. $1-\frac{2}{5}+\frac{4}{25}-\ldots$ to infinity.

687. $4+3+\frac{9}{4}+\ldots$ to infinity.

688. $1+2-\frac{1}{4}+\frac{1}{32}-\ldots$ to infinity.

689. $\frac{1}{10}+\frac{2}{10^2}+\frac{1}{10^3}+\frac{2}{10^4}+\ldots$ to infinity.

690. $1-2x+2x^2-2x^3+\dots$ to infinity.

691. $3\frac{3}{8}+2\frac{1}{4}+1\frac{1}{2}+\dots$ to infinity.

692. $\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{1}{2-\sqrt{2}}+\frac{1}{2}+\dots$ to infinity.

693. $\frac{f}{g}-\frac{f-g}{g^2}x+\frac{f-g}{g^3}x^2-\dots$ to infinity, where x is less than g.

694. $x-y+\frac{y^2}{x}-\frac{y^3}{x^2}+\dots$ to $2n$ terms and to infinity, and find the ratio of these sums.

695. $\frac{a}{x}\sqrt{\frac{3}{2}}+\sqrt{\frac{a}{x}}+\sqrt{\frac{2}{3}}+\dots$ to n terms and also to infinity.

696. $\frac{1}{3}+\frac{1}{6\sqrt{-1}}-\frac{1}{12}+\dots$ to infinity.

697. $1-3x+5x^2-\dots$ to infinity.

698. $\dfrac{1+2x+3x^2+\dots \text{ to infinity.}}{1-2x+3x^2-\dots \text{ to infinity.}}$

699. $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\dots$ to infinity.

700. $\frac{1}{3}+\frac{3}{9}+\frac{5}{27}+\dots$ to infinity.

701. $1+5+13+29+\dots$ to n terms.

702. $1+3+7+15+\dots$ to n terms.

703. $3+6+11+20+\dots$ to n terms.

704. $2+4+\frac{14}{3}+\frac{44}{9}+\frac{134}{27}+\dots$ to n terms.

705. Find the value of the fraction $\dfrac{\cdot 02\dot{7}}{\frac{1}{3}-\frac{1}{2}+\frac{3}{4}-\dots \text{ to infinity.}}$

706. Insert two geometric means between 1 and 100.

707. Insert three geometric means between $\frac{1}{2}$ and 128.

708. Insert four geometric means between 2 and 64.

709. The difference between two numbers is 48, and the arithmetic mean exceeds the geometric by 18. Find the numbers.

710. If the arithmetic mean between a and b is double of the geometric, find the ratio of $a:b$.

711. If a, b, c, d . . be $n+1$ quantities in geometrical progression, prove that the reciprocals of a^2-b^2, b^2-c^2, c^2-d^2, . . . are also in geometrical progression, and find their sum.

712. In a geometric series the $(p+q)$th term $=m$, and the $(p-q)$th term $=n$; find the pth term and also the qth.

713. If a, b and c be in geometrical progression, determine which is greater, $a^2+b^2+c^2$ or $(a-b+c)^2$.

714. If $S_1=1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\ldots$ to infinity,

and $S_2=1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\ldots$ to infinity,

find the ratio $S_1 : S_2$.

715. If a, b, c, d are in geometrical progression, prove that $(a+b+c+d)^2=(a+b)^2+(c+d)^2+2(b+c)^2$.

716. In a geometrical progression, if P and Q denote the pth and qth terms, find the nth.

717. If s = sum of the squares of any two quantities, p = twice their product, and P = the pth power of the sum, prove that

$$P.P^{\frac{1}{2}}.P^{\frac{1}{4}}.P^{\frac{1}{8}}\ldots \text{ to infinity } =s^p+p^2.s^{p-1}+p^3.\frac{p-1}{2}.s^{p-2}$$

$$+p^4.\frac{p-1}{2}.\frac{p-2}{3}.s^{p-3}+\ldots$$

718. Given s_1 and s_2, the sums of the even and odd terms of a geometrical progression of $2n$ terms. Of m arithmetic means inserted between its pth and qth terms, required the rth mean.

719. If P be the product, S the sum, and S_1 the sum of the reciprocals of n quantities in geometrical progression, prove $P^2=\left(\frac{S}{S_1}\right)^n$.

720. The sum of £700 was divided among four persons, whose shares were in geometrical progression; and the difference between the greatest and least was to the difference between the means as 37 to 12. What were their respective shares?

HARMONICAL PROGRESSION.

General Formulæ. If a, b, c, d, . . be consecutive terms of a series in harmonical progression, then will

$$a : c=a-b : b-c \text{ and } b : d=b-c : c-d, \text{ \&c.};$$

also $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$, $\frac{1}{d}$, &c. are in arithmetical progression.

721. Continue in both directions the series 2, 3, 6, and 3, 4, 6. How far can the latter be continued either way?

722. Insert two harmonic means between 2 and 4.

723. Insert two harmonic means between 6 and 24.

724. Insert four harmonic means between 2 and 12.

725. Insert six harmonic means between 1 and 20.

726. Insert n harmonic means between x and y.

727. Given a and b, the first and nth terms respectively of an harmonic series; find the intervening terms.

728. Given x, y, the first and second terms of an harmonical progression; continue the series, and write down the nth term.

729. Show that the arithmetic, geometric and harmonic means between a and b are in continued proportion.

730. If a, b, c are in harmonical progression, determine which is greater, $2b^2$ or a^2+c^2.

731. Having given the mth and nth terms of an harmonical progression, viz. M and N respectively, determine the $(m+n)$th term.

732. There are four numbers of which the first three are in arithmetical progression, the last three in harmonical; prove that the first is to the second, as the third is to the fourth.

733. If y be the harmonic mean between x and z, and x and z be the arithmetic and geometric means respectively between a and b; express y in terms of a and b.

734. If a, b, c be the pth, qth and rth terms respectively of an harmonic series, prove that $(p-q)ab+(r-p)ac+(q-r)bc=0$.

735. If $a^x=b^y=c^z=$&c. and a, b, c, &c. be in geometrical progression, then will x, y, z, &c. be in harmonical progression.

736. Compare the lengths of the sides of a right-angled triangle when the squares described upon them are in harmonical progression.

737. Having given 11, the sum of three numbers in harmonical progression, and 36 their continued product; determine the numbers.

THE PILING OF BALLS AND SHELLS.

General Formulæ. The number of balls in a triangular pile is expressed by $\frac{1}{6}n(n+1)(n+2)$, and in a square pile by $\frac{1}{6}n(n+1)(2n+1)$; where in both cases n denotes the number of balls in one side of the bottom row. Also the number of balls in a rectangular pile is represented by $\frac{1}{6}n(n+1)(3l-n+1)$, where l is the number of balls in the length of the base row, and n the number in the breadth of the same row.

738. Show that $1^3+2^3+3^3+\ldots+n^3=(1+2+3+\ldots+n)^2$.

739. Find the number of balls in a triangular pile, each side of the base row containing 36 balls.

740. Find the number of shells in a square pile, each side of the base row containing 32 shells.

741. Required the number of balls in a rectangular pile, the length and breadth of the base row being 52 and 34.

742. What is the number of balls in an incomplete triangular pile, in which 25 balls form one side of the bottom row, and 13 a side of the top row?

743. What is the number of balls in an incomplete square pile, one side of the bottom containing 44 balls, and of the top 22?

744. An incomplete rectangular pile of 18 courses contains 56 balls in the length, and 38 in the breadth of the bottom row: required the number of balls piled.

745. There are two complete piles of balls, one triangular, the other square; such that the whole number of balls in the former is to the whole number in the latter as 6 is to 11. Required the number of balls in each pile, the bottom row of both having the same number of balls in a side.

746. A complete rectangular pile has the same number of balls in the breadth of its bottom row as form the side of the base of a complete square pile; and the number of balls in the former pile is to the number in the latter as 8 is to 5. Also the number of courses in the rectangular pile added to the number of balls in the length of its bottom row is 29. Required the whole number of balls in each pile.

747. What is the number of shells contained in 7 courses of an incomplete pentagonal pile, when each side of the base row contains 15 shells?

PERMUTATIONS AND COMBINATIONS.

General Formulæ.

1. The number of permutations of n different things, taking r of them at a time, is $n(n-1)(n-2)...(n-r+1)$.

2. The number of permutations of n things, where p of them are of one sort, q of another, r of a third, and so on, all taken together, is $\dfrac{1.2.3...n}{(1.2.3..p)(1.2.3..q)(1.2.3..r)(\&c.)}$.

3. The number of combinations of n different things, taking r of them together, is $\dfrac{n(n-1)(n-2)...(n-r+1)}{1.2.3.....r}$.

748. If ten letters, a, b, c... be combined, 5 and 5 together, in how many of the combinations will a and b occur?

749. How many different sums can be formed with the following coins: a farthing, a penny, a sixpence, a shilling, a crown, a half-sovereign, a guinea, and a moidore?

750. At an election where every voter may vote for any number of candidates not greater than the number to be elected; there are 4 candidates and 3 members to be chosen, in how many ways may a man vote?

751. From a company of 80 men, 9 are draughted off every

night to form a patrol; on how many different nights can a different selection be made; and on how many of these will any particular soldier be engaged?

752. Find all the permutations that can be formed out of the letters of the words (1) *Baccalaureus*, (2) *Mississippi*, (3) *Hippopotamus*, (4) *Museum*, (5) *Commencement*, (6) *Couscoussou*.

753. In how many different ways may the letters of the continued product $a^5.c.e^4.f$ be written?

754. How many combinations will there be of 12 letters, a, b, c, &c., taken 4 and 4 together? In how many of these will a, b and c occur?

755. At a game of cards, 3 being dealt to each person, any one can have 425 times as many hands as there are cards in the pack; required the number of cards.

756. The number of combinations of n things taken 4 at a time is to the number where 2 are taken at a time as 15 to 2. Required the value of n.

757. The number of permutations of n things taken 3 at a time is equal to 6 times the number of combinations of n taken 4 at a time; find n.

758. The number of combinations of n things taken 5 at a time is to the number where 3 are taken at a time as 18 is to 5; find n.

759. The number of permutations of n things taken r at a time is equal to 10 times the number taken $(r-1)$ at a time: and the number of combinations of n things taken r at a time is to the number taken $(r-1)$ at a time as 5 is to 3; required the values of n and r.

760. If $p_2, p_3 \dots p_n$ represent the number of permutations that can be formed out of n quantities taken 2, 3, &c. n together respectively, and $P=p_2p_3 \dots p_n$, prove that

$$P=p_3p_n\{(p_3-p_2)(p_4-p_3)(p_5-p_4)\dots(p_{n-1}-p_{n-2})\}.$$

761. How many words can be formed consisting of 3 consonants and a vowel, in a language which contains 19 consonants and 5 vowels?

762. Find the number of words which can be formed by taking the 24 letters of the alphabet, 6 at a time, each word containing 2 vowels.

763. Out of 17 consonants and 5 vowels, how many words can be formed, each consisting of 2 consonants and 1 vowel?

764. The total number of combinations of $2n$ things divided by the total number of combinations of n things is equal to 65; find n.

765. There are 4 regular polyhedrons marked in the manner of dice, and the numbers of their faces are 3, 6, 8, 12 respectively; taking all of them together, how many different throws can possibly be made?

766. The number of combinations of n things taken p together is to the number taken $p+2$ together as a is to b; if a, b, p represent 5, 18, 3 respectively, find n.

767. Find the number of permutations of n things taken r at a time, with repetitions, i. e. allowing quantities which recur to be combined as if they were different.

768. Find the number of different combinations of n things taken 1, 2, 3 . . . n together; of which n things, p are of one sort, q of another, r of a third, and so on, subject to the condition that $p+q+r+\ldots=n$.

769. When the number of combinations of $2n$ things taken r at a time is the greatest possible; find r.

770. Express the number of combinations of $n+1$ things taken r at a time, in terms of the number of combinations of n things taken r at a time, and also $r-1$ at a time.

771. Investigate a formula for expressing the number of homogeneous products which can be formed of n things of r dimensions; and determine the number of terms in the expansion of $(a+b+c+d)^{10}$.

BINOMIAL AND MULTINOMIAL THEOREMS.

General Formulæ. In the expansion of $(a+x)^n$ the $(r+1)$th term is represented by $\frac{n(n-1)\ldots(n-r+1)}{1.2\ldots.r}a^{n-r}.x^r$; where n is positive or negative, and integral or fractional.

In the expansion of $(a+bx+cx^2+\ldots+kx^t)^n$ the general term is represented by

$$\frac{1.2.3\ldots n}{(1.2.3\ldots p)(1.2.3\ldots q)(1.2.3\ldots r)\,\&c.}a^p.b^q.c^r,\&c.\,x^{q+2r+\ldots},$$

where $p+q+r+\ldots=n$, and $q+2r+\ldots=m$, if m denote the index of x in the general term.

772. Expand $(a+x)^9$, $(a-x)^7$, $(2x-3y)^4$, and $\left(5-\frac{x}{6}\right)^6$.

773. Expand $(1+x)^{\frac{m}{n}}$, $(1-x)^{\frac{5}{6}}$, $\left(a+\frac{4}{3}x\right)^{\frac{3}{4}}$, and $\left(\frac{2}{3}x-\frac{3}{2}y\right)^{\frac{2}{3}}$.

774. Expand $(c-x)^{-t}$, $(a+h)^{-s}$, $(1-x)^{-\frac{1}{2}}$, $(a^2-x^2)^{-\frac{1}{3}}$, and $(a+x)^{-2}$.

775. Expand $(a+x\sqrt{-1})^n$, $(b-y\sqrt{-1})^8$, and $(h+k\sqrt{-1})^7$.

776. Expand $\frac{1}{\sqrt[3]{ax-x^2}}$, $\frac{1}{(\sqrt[3]{a}-\sqrt[3]{x})^6}$, and $\frac{1}{a+a^{\frac{2}{3}}x^{\frac{1}{3}}}$.

777. Required the coefficient of x^5 in the expansion of $(a-x)^9$.

778. Required the coefficient of x^9 in the expansion of $(5a^3-4x^3)^7$.

779. Required the coefficient of x^{12} in the expansion of $(a^5-b^3x^2)^{\frac{5}{2}}$.

780. Required the 6th term of the expansion of $\sqrt{4a^2cx-3c^{\frac{2}{3}}y^{\frac{1}{3}}}$.

781. Required the 5th term of the expansion of $(ax-by)^{-10}$.

782. Required the 6th term of the expansion of $(ax-bx^2\sqrt{-1})^{\frac{3}{5}}$.

783. Required the 8th term of the expansion of $(-s+t\sqrt{-1})^{-\frac{5}{7}}$.

784. Required the greatest term in the expansions of

$$\left(\frac{\sqrt{x}}{2}-\frac{\sqrt[3]{y}}{3}\right)^{\frac{2}{3}} \text{ and of } \left(1+\frac{5}{6}\right)^{\frac{3}{2}}.$$

785. Required the middle terms of $(a^n-x^n)^{10}$ and of $(1+x)^{2n}$, which latter may be made to assume the form $\frac{1.3.5\ldots(2n-1)}{1.2.3\ldots n}(2x)^n$.

786. Write down the $(r+1)$th terms of $(3a+2x)^{-\frac{1}{5}}$ and of $(xy-\sqrt{3yz})^{\frac{5}{4}}$.

787. Expand $\left(x+\frac{1}{x}\right)^{2n}$ and $\left(x-\frac{1}{x}\right)^{2n+1}$, bracketing in pairs the terms equidistant from the beginning and end of each expansion.

788. Prove that

$$(a-b)^n=a^n\left\{1-n\cdot\frac{b}{a-b}+\frac{n(n+1)}{1.2}\left(\frac{b}{a-b}\right)^2-\&c.\right\}.$$

789. Prove that

$$\left(\frac{1+2x}{1+x}\right)^n=1+n\cdot\frac{x}{1+2x}+n\cdot\frac{n+1}{2}\cdot\left(\frac{x}{1+2x}\right)^2+\&c.$$

790. Prove that $\frac{1+x}{1-x}=1+n\cdot\frac{2x}{1+x}+n\cdot\frac{n+1}{2}\left(\frac{2x}{1+x}\right)^2+\&c.$

Find the value of the following series:—

791. $1-\frac{1}{2}\cdot\frac{1}{2}+\frac{1.3}{2.4}\cdot\frac{1}{2^2}-\frac{1.3.5}{2.4.6}\cdot\frac{1}{2^3}+\&c.$ *ad infinitum.*

792. $1-2n+3n\cdot\frac{n+1}{2}-4n\cdot\frac{n+1}{2}\cdot\frac{n+2}{3}+\&c.$ *ad infinitum.*

793. $a-(a+b)n+(a+2b)n\cdot\frac{n-1}{2}-(a+3b)n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}+\&c.$

794. $1+\frac{n}{2}+\frac{n}{3}\cdot\frac{n-1}{.2}+\frac{n}{4}\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}+\ldots$

795. If the coefficients of the expansion of $(a-x)^m$ be multiplied by 1^n, 2^n, 3^n, &c., in order, the result will $=0$ when m is greater than n.

796. If the coefficients of $(a-x)^n$ be multiplied by 1.2, 2.3, 3.4, &c., in order, the result will $=0$.

797. If x be greater than a, show that the sum of all the terms of the expansion of $(x+a)^n$, after the first two, is less than $(2^n-n-1).ax^{n-1}$.

798. If $z^2+z+1=0$, show that the sum of those terms in the expansion of $(1+x)^n$, in which the index of x is a multiple of 3, is $=\frac{1}{3}\left\{\left(1+\frac{x}{z}\right)^n+(1+x)^n+(1+xz)^n\right\}$.

799. Find the sum of the squares of the coefficients in the expansion of $(1+x)^n$, when n is a positive integer.

800. Find the sum of the products of every two consecutive coefficients in the expansion of $(1+x)^n$, n being a positive integer.

801. Prove that 4 times the product of the sums of the odd and even terms of the expansion of $(a+x)^n$ is $=(a+x)^{2n}-(a-x)^{2n}$.

802. Find the coefficient of x^n in the expansion of

$$(1+x+2x^2+3x^3+\ldots)^2.$$

803. If a, b, c, d be any consecutive coefficients of an expanded binomial, then will $(bc+ad)(b-c)=2(ac^2-db^2)$.

804. If the $(r+1)$th term of the expansion of $(1+x)^n$ when n is a positive integer be equal to the $(r+3)$th term, it is required to find r.

805. If $s=$ sum of two quantities, $p=$ their product, and $q=$ quotient, prove that $p^2=s^4\left(q^2-4q^3+\frac{4.5}{1.2}.q^4-\frac{4.5.6}{1.2.3}q^5+\ldots\right)$ and the sum of the nth powers of the two quantities $=s^n-np.s^{n-2}+\frac{n(n-3)}{1.2}.p^2.s^{n-4}-\ldots$.

806. Given P and Q, the coefficients of the pth and qth terms of the expansion of $(a+x)^n$; find n.

807. If n_r be the coefficient of the $(r+1)$th term of $(1+x)^n$ expanded, show that $(n+p)_r=n_r+n_{r-1}p_1+n_{r-2}p_2+\ldots+n_1p_{r-1}+p_r$.

808. If m_r be the coefficient of the $(r+1)$th term of the expansion of $(1-x)^{-m}$, show that $m_r+(m+1)_{r-1}=(m+1)_r$.

809. If $x=a(1+h)$, where h is a very small fraction; find the value of $\sqrt{2ax-x^2+\left(\frac{b}{a}\right)^2x^2}\div\sqrt{a^2+b^2}$.

810. Find the value of $\left(\frac{1}{x^2-x}+\frac{1}{4x^2}\right)^{\frac{1}{2}}-\frac{1}{2x}$ when $x=0$.

811. Find the value of $\{a-\sqrt[n]{a^n-x^n}\}\div x^n$ when $x=0$.

812. Find the value of $\{x^2-a^2)^{\frac{3}{2}}+(x-a)\}\div\{(1+x-a)^3-1\}$ when $x=a$.

813. Find the value of $\frac{(a^2x+7a^3)^{\frac{1}{3}}-(2ax+2a^2)^{\frac{1}{2}}}{(a^2x^6+15a^5x^3)^{\frac{1}{4}}-(15a^4x^6+17ax^9)^{\frac{1}{5}}}$ when $x=a$.

814. If $x \backsim 1$ be a very small quantity, show that

$$\frac{mx^m - nx^n}{m-n} = x^{m+n} \text{ nearly.}$$

815. Required the coefficient of x^4 in the expansion of $(1+x+x^2)^5$.

816. Required the term involving $a^2b^2c^3$ in the expansion of $(a+b+c)^7$.

817. Find the coefficient of x^5 in the expansion of $(a+bx+cx^2+dx^3)^6$.

818. In the expansion of $(2+4x-3x^2+x^3)^7$, find the coefficient of x^8.

819. In the expansion of $(a-bx-cx^2)^{\frac{m}{n}}$, find the coefficient of x^5.

820. Required the coefficient of $a^{n-5}.b^2.c^3$ in the expansion of $(a+b+c)^n$.

821. Find the term involving $b^2.c^3.e^4.f$ in the expansion of $(a+2b+3c+4d+5e+6f)^{10}$.

822. Find the coefficient of x^5 in $(1+2x+3x^2+4x^3+\ldots)^6$ expanded.

823. Find the coefficient of x^6 in $(a+bx+cx^2+\ldots)^n$ expanded.

824. Expand $(1+x+x^2+x^3+\ldots)^{\frac{1}{2}}$.

825. Expand $(1+x+x^2+x^3+\ldots)^2$.

826. Expand $\left(1-\frac{x}{2}+\frac{2x^2}{3}-\frac{3x^3}{4}+\ldots\right)^{-2}$.

827. Expand $\left(1-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots\right)^2$.

828. Required the coefficient of x^r in $(1+2x+3x^2+\ldots)^2$ expanded.

829. What is the number of terms in the expansion of $(a+b+c)^n$?

830. What is the number of terms in the expansion of $(a+bx+cx^2+dx^3)^4$?

831. Find the middle term in the expansion of $(1+x-x^2)^n$.

832. What is the coefficient of the middle term in the expansion of $(2-5x-7x^2+x^3+3x^4)^5$?

INDETERMINATE COEFFICIENTS.

833. Resolve $\frac{6x^2-4x-6}{(x-1)(x-2)(x-3)}$ into partial fractions.

834. Find the fractions which, when united by addition, shall give $\frac{x^2}{(x^2-1)(x-2)}$.

835. Find the fractions whose sum is $\frac{1+5x+3x^2}{(1+x)^2(1+2x)^2}$.

836. Expand $\frac{3+2x}{5+7x}$ in a series of ascending powers of x.

837. Resolve $\frac{x^2+hx+k}{(x-a)(x-b)(x-c)}$ into partial fractions.

838. Expand $\frac{1+2x}{1-x-x^2}$ in a series of ascending powers of x.

839. Resolve $\frac{x^2+x+1}{x^3(x^2+1)^2}$ into partial fractions.

840. Find three fractions whose sum is $\frac{1}{x^4-a^4}$.

841. If $x=y-\frac{y^2}{2}+\frac{y^3}{3}-$&c..., find y in terms of x.

842. If $y=x+\frac{x^3}{1.2.3}+\frac{3.x^5}{1.2.4.5}+\frac{3.5.x^7}{1.2.4.6.7}+\dots$, find x in terms of y.

843. If $y=ax+bx^2+cx^3+dx^4+\dots$, express x in a series of ascending powers of y.

844. If $ny^3-xy=n$, expand y in a series of ascending powers of x.

845. If $y^3+x=3y$, expand y in a series of ascending powers of x.

---

846. Find the sum of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots$ to n terms, and also to infinity.

847. Find the sum of $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+\dots$ to n terms, and to infinity.

848. Find the sum of $\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\dots$ to infinity.

849. Find the sum of $\frac{1}{\sqrt{2}(1+\sqrt{2})}+\frac{1}{(1+\sqrt{2})(2+\sqrt{2})}+\frac{1}{(2+\sqrt{2})(3+\sqrt{2})}+\dots$ to infinity.

850. Find the sum of $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\dots$ to n terms, and to infinity.

851. Find the sum of $1^2+6^2+15^2+20^2+15^2+6^2+1^2$.

852. Sum the series $1^2+11^2+55^2+165^2+330^2+462^2+462^2+330^2+165^2+55^2+11^2+1^2$.

853. Sum the series $1.9+9.36.+36.84+84.126+126^2+126.84+84.36+36.9+9.1$.

SCALES OF NOTATION.

854. Express the denary number 790158 in the septenary scale.

855. Express the quinary number 34402 in the quaternary scale.

856. Transform 6587 and 3907 from the common scale to the duodenary, and find the product of the results.

857. Divide 14332216 by 6541 in the septenary scale.

858. Divide 95088918 by *tt*4 in the duodenary scale.

859. Find the radix of that scale in which 40501 is equivalent to the denary number 5365.

860. The number 124 in the denary scale is expressed by 147 in another scale; required the radix of the latter.

861. Any number is divisible by 8, if the number consisting of the last 3 digits in order be divisible by 8.

862. Any number consisting of an even number of digits, in a system whose radix is r, is divisible by $r+1$, if the digits equidistant from each end are the same.

863. If N, N' be any two numbers in the denary scale, composed of the same digits differently arranged, prove that $N \backsim N'$ is divisible by 9.

864. Extract the square root of 25400544 in the senary scale, and of 32*e*75721 in the duodenary.

865. Which of the weights 1, 3, 3^2, &c. pounds must be selected to weigh 304 pounds?

866. If $a \div b$ be an irreducible fraction, and b be any number except 9 or 3, show that when it is converted into a recurring decimal, the period will be divisible by 9, and the sum of the remainders a multiple of 6.

867. If $a+br+cr^2+\ldots+kx^n$ represent a number of $n+1$ digits; find the sum of all the numbers that can be formed by placing the digits in all possible positions.

868. There is a number consisting of 3 digits in geometrical progression; the number is to the sum of its digits $=124:7$; and if 594 be added to it, the digits will be inverted: find the number.

869. If the number expressed by the last n digits of a number be divisible by 2^n, the number itself is divisible by 2^n.

870. Transform 8978 from a local value 11, and 3256 from a local value 7, to a system in which the local value is 12; and multiply the numbers together in this system.

871. Find the value of the circulating senary fraction 45·2534534, &c.

PROPERTIES OF NUMBERS.

872. If n be a whole number, prove that n^3+5n is divisible by 6.

873. No number can be a square which has any one of the numbers 2, 3, 7, 8 for its last digit.

874. If m be any odd square number greater than 1, prove that $(m+3)(m+7)$ is divisible by 32.

875. If n be any positive whole number, show that n^5-5n^3+4n is divisible by 120.

876. If n be any whole number, one of the three numbers n^2, n^2+1, n^2+2 is divisible by 5.

877. If n be a prime number, and a prime to n, prove that $(a^n-1)\div n$ is an integer.

878. Decompose 9147600 into its prime factors; and find the multiplier which will make it a perfect cube.

879. If m be a prime number, and N a number not divisible by m, then will $N^{m-1}-1$ be divisible by m.

880. If m be a prime number, then will $1.2.3...(m-1)+1$ be divisible by m.

881. If n be an odd number $n(n^4-1)\div 48$ is an integer.

882. If an odd and an even square number be added together, and the sum be also a square, the even square is a multiple of 16.

883. Find the number of divisors of 2160, and also their sum.

884. Prove that $(n+1)(n+2)(n+3)$, &c. to n factors is $=2^n(1\times 3\times 5\times$ &c. to n factors).

885. Prove that the difference of the squares of any two odd numbers is divisible by 8; and that some term of the progression $r, r^2, r^3, \ldots r^{n-1}$ diminished by unity, is divisible by n, r being an integer not divisible by n.

LOGARITHMS.

MULTIPLICATION BY LOGARITHMS.

886. Multiply 24·13 by 6·052.
887. Multiply 49·51 by 283·605.
888. Multiply 5·281925 by 4·375921.
889. Multiply ·007461 by ·3351767.
890. Multiply 20·192248 by 634·47.
891. Multiply ·0700379 by ·0086752.
892. Multiply 2487492 by ·006988964.
893. Multiply 864665·2 by 8·097466.
894. Multiply ·00087214 by ·001963.
895. Multiply ·034632 by ·397302.
896. Find the continued product 4·002 × 608·27 × ·0425839.
897. Find the continued product 4·697 × 3·2157 × ·9483 × ·0305.

DIVISION BY LOGARITHMS.

898. Divide 35274 by 5678.
899. Divide 48·25 by 634·87.
900. Divide ·07425 by ·008352.
901. Divide ·9649 by 35·0583.
902. Divide 11 by ·3929.
903. Divide ·26439 by ·28629.
904. Divide ·001048869 by ·00471698.
905. Divide ·0697565 by ·9975641.

INVOLUTION BY LOGARITHMS.

906. Find the square of 35·7924.
907. Find the cube of 5·0008562.
908. Find the fourth power of ·05632.
909. Raise 1·0043 to the 365th power.
910. Find the 65th power of 3.
911. Find the ·005th power of ·005.
912. Find the ·0008th power of ·2846.

EVOLUTION BY LOGARITHMS.

913. Find the square root of 3852.
914. Find the cube root of 199586·251.
915. Find the 20th root of 5.
916. Find the 365th root of 1·045.
917. Find the cube root of ·00052653.
918. Find the fifth root of ·0856329.
919. Find the 19th root of ·00123456.
920. Find the ·08th root of ·08.
921. Find the ·0065th root of ·16235.

MISCELLANEOUS EXAMPLES IN LOGARITHMS.

922. Find a fourth proportional to 8352, 3·69, and 30·57.

923. Find a fourth proportional to $\sqrt{8{\cdot}37}$, $\sqrt[3]{\frac{2}{15}}$, and ·0084321.

924. Find a third continued proportional to

$\sqrt[3]{\frac{24}{{\cdot}017}}$, and $\sqrt[7]{{\cdot}005234}$.

925. Find a mean proportional between $\frac{\sqrt[7]{15{\cdot}92}}{{\cdot}00526}$ and $\frac{\sqrt[3]{{\cdot}0182}}{(196)^5}$.

926. Find the value of $\left(\frac{35}{18634}\right)^{\frac{3}{7}}$.

927. Find the value of $\sqrt[11]{\frac{15}{16}\sqrt[7]{{\cdot}018}}$.

928. Find the value of $\sqrt[100]{\frac{13659 \times (8{\cdot}256)^{10}}{\sqrt[3]{(1{\cdot}86)^5}}}$.

929. Find the value of $\frac{\frac{1}{6}\sqrt[3]{\frac{13}{14}} \times {\cdot}004\sqrt[7]{846}}{13\sqrt{152} \div {\cdot}34\sqrt[9]{{\cdot}186}}$.

930. Find the value of $\frac{(23{\cdot}86)^{\frac{2}{5}} - (84{\cdot}28)^{\frac{1}{3}} + \sqrt[7]{{\cdot}001} + 1}{15\frac{2}{19} + \frac{1}{15}\sqrt[9]{28{\cdot}64}}$.

931. Find the value of $\frac{(1{\cdot}025)^{13} - 1}{(1{\cdot}025)^{13} + 1}$.

932. Find the value of (£32 16*s.* 7¾*d.*) × $(1{\cdot}015)^{50}$.

LOGARITHMIC EQUATIONS.

933. Given $a^x=b$, to find x.
934. Given $20^x=100$, to find x.
935. Given $2^x=769$, to find x.
936. Given $a^{bx}=c$, to find x.
937. Given $a^{mx}.b^{nx}=c$, to find x.
938. Given $\left(\frac{5}{4}\right)^x=54\frac{1}{2}$, to find x.
939. Given $\left(\frac{\sqrt{7}}{\sqrt[3]{9}}\right)^{2x}=2^{\frac{1}{3}}.6^{\frac{1}{2}}$, to find x.
940. Given $(2^3)^x\times(3^2)^x=4{\cdot}9$, to find x.
941. Given $\left(\frac{3}{673}\right)^{-\frac{1}{x}}=1{\cdot}75$, to find x.
942. Given $a^x.b^y=m$, and $x:y=c:d$, to find x and y.
943. Given $a^{x^x}+b^{y^y}=c$, and $x:y=m:n$, to find x and y.
944. Given $a^{4x}+a^{2x}=a6^{6x}$, to find x when $\log a=1$.
945. Given $a^x+a^{-x}=b$, to find x.
946. Given $x^y=y^x$ and $x^a=y^b$, to find x and y.
947. Given $3^{2x}.5^{3x-4}=7^{x-1}.11^{2-x}$, to find x.
948. Given $(a^4-2a^2b^2+b^4)^{x-1}=\frac{(a-b)^{2x}}{(a+b)^2}$, to find x.

---

949. Given log 2=·301030 and log 3=·477121, find the logarithms of 6, 15, ·05, 36, 5·4, and 1800.

950. Given log 2=·301030 and log 7=·845098, find the logarithms of 35, 70, 17·5, ·875, 280, and 6860.

951. Given log 18=1·255272 and log 25=1·397940, find the logarithms of 2, 3, ·16, 450, ·075, and 3·75.

952. If log 67833=4·8314410 and log 67832=4·8314346, find the logarithm of 678328.

953. If log 33819=4·5291608 and log 33818=4·5291479, find the logarithm of 338189.

For Examples in Interest requiring the application of Logarithms, see Arithmetic, page 15.

GEOMETRICAL DEDUCTIONS.

The references at the end of each example are (1) to the propositions in Euclid, and (2) to those in the Geometry of Cape's Course (2nd edition), on which the proof chiefly depends.

1. The straight line which bisects the vertical angle of an isosceles triangle, bisects the base, and is perpendicular to it. (Euclid, i. 4. Cape, i. 6.)

2. The straight line which bisects the base of an isosceles triangle, is perpendicular to it, and bisects the vertical angle. (Euclid, i. 8. Cape, i. 15.)

3. If the line bisecting the vertical angle of a triangle be perpendicular to the base, the triangle is isosceles. (Euclid, i. 26. Cape, i. 7.)

4. If the line bisecting the vertical angle of a triangle, also bisect the base, the triangle is isosceles. (Euclid, i. 4, 16. Cape, i. 6, 10.)

5. The lines bisecting an internal, and the adjacent external angle of a triangle, are at right angles to each other. (Euclid, i. 13. Cape, i. 2.)

6. From two given points, to draw two straight lines to meet in a given straight line, and to make equal angles with it. (Euclid, i. 12, 15. Cape, i. 5, and prob. 3.)

7. Of any two straight lines, that may be drawn from two given points without a given line, to meet in that line, the sum is the least, when they make equal angles with the line. (Euclid, i. 21. Cape, i. 13.)

8. On a given straight line, to describe a square, of which it shall be the diagonal. (Euclid, i. 10, 11. Cape, probs. 1, 2.)

9. To trisect a given finite straight line. (Euclid, i. 9, 28. Cape, prob. 5, and i. 20.)

10. The difference between any two sides of a triangle is less than the third side. (Euclid, i. 16, 19. Cape, i. 10, 11.)

11. If any two sides of a triangle be bisected, and from the middle points, perpendiculars be drawn to the sides, the straight lines joining the point of intersection with the three angular points of the triangle are equal to one another. (Euclid, i. 4. Cape, 1, 6.)

12. If any two angles of a triangle be bisected by straight lines, which meet in a point, the three perpendiculars drawn from this point to the sides of the triangle are equal to one another. (Euclid, i. 26. Cape, i. 7.)

13. The three perpendiculars drawn from the middle points of the sides of a triangle, intersect in one point. (Euclid, i. 4, 8. Cape, i. 6, 15.)

14. The three straight lines bisecting the angles of a triangle, intersect in one point. (Euclid, i. 26. Cape, i. 7.)

15. The three straight lines joining the angular points of a triangle with the middle points of the opposite sides, intersect in one point. (Euclid, i. 31, 26, 34. Cape, prob. 6, and i. 7, 26.)

16. In a right-angled triangle, the line drawn from the right angle to the middle point of the hypothenuse is equal to half the hypothenuse. (Euclid, i. 26, 34. Cape, i. 7, 26.)

17. In a right-angled triangle, the angle contained by the line bisecting the right angle, and the line drawn perpendicular to the hypothenuse, is equal to half the difference of the two acute angles of the triangle. (Euclid, i. 32. Cape, i. 23.)

18. If each of the equal angles of an isosceles triangle be equal to half the vertical angle, and from one of them a perpendicular be drawn to the base, meeting the opposite side produced, then will the part produced, the perpendicular, and the remaining side, form an equilateral triangle. (Euclid, i. 32. Cape, i. 23.)

19. The quadrilateral figure, whose diagonals mutually bisect each other, is a parallelogram. (Euclid, i. 4, 27. Cape, i. 6, 19.)

20. The parallelogram, whose diagonals are equal, is rectangular. (Euclid, i. 8, 29. Cape, i. 15, 20.)

21. The parallelogram, whose diagonals intersect at right angles, is equilateral. (Euclid, i. 4. Cape, i. 6.)

22. Through a given point to draw a straight line which shall make equal angles with two given straight lines. (Euclid, i. 10, 12. Cape, prob. 1, 3.)

23. The area of a rhombus equals half the rectangle contained by the diagonals. (Euclid, i. 41. Cape, iii. 52.)

24. From a given point between two given straight lines, to draw a straight line, which shall be terminated by the given straight lines, and bisected by the given point. (Euclid, i. 12, 26. Cape, prob. 3, and i. 7.)

25. If in the sides of a square, four points be taken, at equal distances from the four angular points taken in order; the figure contained by the straight lines which join them shall also be a square. (Euclid, i. 4, 16, 32. Cape, i. 6, 10, 24.)

26. If the sides of any hexagon be produced to meet, the angles formed by these lines are together equal to four right angles. (Euclid, i. 32. Cape, i. 24.)

27. If the sides of any pentagon be produced to meet, the angles formed by these lines are together equal to two right angles. (Euclid, i. 32. Cape, i. 24.)

28. If the sides of any polygon be produced to meet, the angles

formed by these lines are together equal to twice as many right angles wanting eight as the polygon has sides. (Euclid, i. 32. Cor. 2. Cape, i. 26.)

29. If from any angle of a triangle, a straight line be drawn to the middle point of the opposite side, the sum of the squares of the sides containing this angle shall be equal to twice the square of the bisecting line, together with twice the square of half the bisected side. (Euclid, ii. 12, 13. Cape, iii. 65.)

30. In a parallelogram, the sum of the squares of the sides is equal to the sum of the squares of the diagonals. (Euclid, ii. 13. Cape, iii. 65. Cor.)

31. If from the angles of a triangle, lines be drawn bisecting the opposite sides, four times the squares of these lines is equal to three times the squares of the sides of the triangle. (Euclid, ii. 13. Cape, iii. 63.)

32. In any triangle ABC, if BP, CQ be drawn perpendicular to CA, BA, produced if necessary, then shall $BC^2 = AB \times BQ + AC \times CP$. (Euclid, ii. 12 or 13. Cape, iii. 63 or 64.)

33. If ABC be an isosceles triangle $(B = C)$, and if CD be drawn perpendicular to AB, then shall $CD^2 = BD^2 + 2BD \times DA$. (Euclid, ii. 13. Cape, iii. 63.)

34. If two circles touch, either externally or internally, and from the point of contact two chords be drawn, meeting the circumferences, the chords of the intercepted arcs will be parallel. (Euclid, iii. 32. Cape, ii. 48.)

35. If two chords intersect in a circle, the difference of their squares is equal to the difference of the squares of the difference of the segments. (Euclid, ii. 8, and iii. 35. Cape, iii. 61, 77.)

36. If two chords be drawn from any point of a circle, and upon these chords, as diameters, two other circles be described, the three points of intersection of these three circles will be in the same straight line. (Euclid, iii. 31. Cape, ii. 47.)

37. A common tangent is drawn to two circles, which touch externally; if a circle be described on that part of it which lies between the points of contact, as diameter, this circle will pass through the common point of contact of the two circles, and be touched by the line which joins their centres. (Euclid, iii. 18. Cape, ii. 38.)

38. If from any point without a circle, two straight lines be drawn, making equal angles with the line through the centre, they will cut off equal segments from the circle. (Euclid, iii. 3. Cape, ii. 35.)

39. DF is a straight line touching a circle, and terminates by AD, BF, tangents at the extremities of any diameter AB; show that the angle subtended by DF at the centre is a right angle. (Euclid, iii. 18. Cape, ii. 38.)

40. If the chord of a quadrant be made the diameter of a semicircle, and from its extremities two straight lines be drawn to any

point in the arc of the semicircle, the segment of the greater line between the two arcs is equal to the less line. (Euclid, iii. 22, 31. Cape, ii. 46, 47.)

41. Show that there can be six equal circles placed around a given circle of the same diameter, so as to touch each other and the given circle. (Euclid, iii. 12. Cape, ii. 40.)

42. If on two lines containing an angle segments of circles be described, containing angles equal to it, the straight lines being produced, shall touch the circles of which they are segments. (Euclid, iii. 16. Cape, ii. 38.)

43. Of all lines which touch the interior and are bounded by the exterior of two circles which touch internally, the greatest is that which is parallel to the common tangent. (Euclid, iii. 7, 15. Cape, ii. 37.)

44. If two circles cut each other, the line joining their centres shall bisect their common chord. (Euclid, iii. 3. Cape, ii. 35.)

45. If two circles cut each other, and from either point of intersection diameters be drawn, the lines joining the extremities of these diameters will pass through the other point of intersection. (Euclid, iii. 31. Cape, ii. 47.)

46. If a common tangent be drawn to any number of circles which touch each other internally, and from any point of this tangent as a centre, a circle be described, cutting the other circles; and if from this centre, lines be drawn through the intersections of the circles, the segments of the lines within each circle shall be equal. (Euclid, iii. 36. Cape, iii. 79.)

47. To draw two straight lines from two given points to meet in a line, given in position, and which shall contain a right angle. (Euclid, iii. 31. Cape, ii. 47.)

48. To find a point, such that the tangents drawn from it, to touch two given circles, may contain a given angle. (Euclid, iii. 16. Cape, ii. 38.)

49. If a straight line touch the interior of two concentric circles, and be terminated by the exterior one, it will be bisected by the point of contact. (Euclid, iii. 18, 3. Cape, ii. 38, 35.)

50. If from the extremities of any diameter of a given circle, perpendiculars be drawn to any chord of the circle, or the chord produced, that is not parallel to the diameter, the less perpendicular shall be equal to the segment of the greater contained between the circumference and the chord. (Euclid, iii. 31. Cape, ii. 47.)

51. If a tangent to a circle be parallel to a chord, the point of contact is the middle point of the arc cut off by the chord. (Euclid, iii. 32. Cape, ii. 49.)

52. Any two parallel lines meeting a circle cut off equal arcs. (Euclid, iii. 21. Cape, ii. 49.)

53. If two chords of a circle intersect at right angles, the sum of the squares of the four segments equals the square of the diameter. (Euclid, iii. 31. Cape, ii. 47.)

54. If two chords of a circle intersect at right angles, and thus cut off four arcs, the sum of any two opposite arcs equals the sum of the other two. (Euclid, iii. 31. Cape, ii. 47.)

55. AB is the diameter of a circle, MN a chord parallel to AB; in AB take any point P, and join PM, PN, then shall the sum of the squares of PM and PN equal the sum of the squares of AP and BP. (Euclid, iii. 31, 35. Cape, ii. 47, and iii. 77.)

56. If a circle be described on the radius of another circle, as its diameter, any straight line drawn from the common point of contact, and terminated by the outer circumference, is bisected by the inner one. (Euclid, iii. 3, 31. Cape, ii. 35, 47.)

57. To draw a straight line which shall touch two given circles. (Euclid, iii. 17. Cape, prob. 21.)

58. If from any point without a circle two lines be drawn touching it, the angle contained by these lines is double the angle contained by the chord joining the points of contact, and the diameter drawn from one of these points. (Euclid, iii. 18. Cape, ii. 38.)

59. To divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square of a given line, which is less than the line to be divided. (Euclid, iii. 16, 36. Cape, ii. 38 and iii. 79.)

60. To draw a straight line, which shall touch a given circle, and make with a given straight line an angle equal to a given angle. (Euclid, iii. 16. Cape, ii. 38.)

61. To describe a circle which shall pass through a given point, and touch a given straight line in a given point. (Euclid, iii. 16. Cape, ii. 38.)

62. To describe a circle which shall pass through a given point, have a given radius, and touch a given straight line. (Euclid, iii. 16. Cape, ii. 38.)

63. To describe a circle which shall pass through a given point, and touch a given circle in a given point. (Euclid, iii. 17. Cape, prob. 21.)

64. To describe a circle which shall touch a given straight line in a given point, and also touch a given circle. (Euclid, iii. 16. Cape, ii. 38.)

65. To describe a circle which shall touch a given circle in a given point, and also touch a given straight line. (Euclid, iii. 16. Cape, ii. 38.)

66. To describe a circle which shall have a given radius, and its centre in a given straight line, and shall also touch another straight line inclined at a given angle to the former. (Euclid, iii. 16. Cape, ii. 38.)

67. If an equilateral triangle be inscribed in a circle, and chords be drawn from its angles to any one point in the circumference, that chord which falls within the triangle shall be equal to the sum of the other two. (Euclid, iii. 21. Cape, ii. 45.)

68. From the obtuse angle of a triangle, to draw a straight line to the base, such that it may be a mean proportional between the segments of the base. (Euclid, iv. 5. Cape, prob. 24.)

69. To divide a right angle into five equal parts. (Euclid, iv. 10. Cape, prob. 27.)

70. To inscribe a circle in a given rhombus. (Euclid, i. 12, and iii. 16. Cape, prob. 3, and ii. 38.)

71. To inscribe a circle in a given segment of a circle. (Euclid, i. 9, and iii. 16. Cape, prob. 5, and ii. 38.)

72. The area of an equilateral triangle inscribed in a circle is equal to one-fourth of that described about it. (Euclid, vi. 2. Cape, iii. 66.)

73. The area of the inscribed square is equal to one-half of that of the circumscribed. (Euclid, i. 34. Cape, iii. 52.)

74. The area of the inscribed regular hexagon is equal to three-fourths of that of the circumscribed. (Euclid, vi. 20. Cape, iii. 75.)

75. If ABC be an isosceles triangle, and DE be drawn parallel to the base BC, then shall $BE^2 = BC \times DE + CE^2$. (Euclid, vi. 2. Cape, iii. 66.)

76. ABC is an equilateral triangle, E any point in AC; in BC produced take $CD = CA$, $CF = CE$, and let AF, DE intersect in H; prove that $HC(AC + EC) = AE \times EC$. (Euclid, vi. 2. Cape, iii. 66.)

77. If a triangle ABC have a right angle C, and AD be drawn bisecting the angle A, and meeting BC in D, then shall $AC^2 : AD^2 = BC : 2BD$. (Euclid, vi. 3. Cape, iii. 67.)

78. Prove also that in the same triangle $2AC^2 : AC^2 - CD^2 = BC : CD$. (Euclid, vi. 3. Cape, iii. 67.)

79. APB is the quadrant of a circle, SPT a line touching it at P; C is the centre, PM perpendicular to CA; prove that triangle SCT : triangle ACB = triangle ACB : triangle CMP. (Euclid, vi. 8. Cape, iii. 72.)

80. In any triangle ABC, if AE be drawn from the angle A in any direction, and BE, CF be drawn perpendiculars to it from B and C, and EG, FG be drawn to the point of bisection G of BC; then shall GE equal GF. (Deduction 16.)

81. If perpendiculars be drawn from the extremities of the base of a triangle to a straight line, which bisects the angle opposite to the base, the area of the triangle is equal to the rectangle contained by either of the perpendiculars, and the segment of the bisecting line that is between the angle and the other perpendicular. (Euclid, vi. 4. Cape, iii. 69.)

82. If from any point within an equilateral triangle, perpendiculars be drawn to the sides, the sum of these perpendiculars is equal to the perpendicular drawn from either of the angles to the opposite side. (Euclid, vi. 1.)

83. To find two straight lines which shall be arithmetic means between two given straight lines. (Deduction 9.)

84. To divide a given straight line AB by two points of division C and D, so that AC, AD, AB may be in harmonical proportion. (Euclid, vi. 4. Cape, iii. 69.)

85. If through any point D of a straight line AD which bisects a given angle BAC, a straight line GFDE be drawn, meeting AG (which is drawn perpendicular to AD), AB and AC respectively in G, F, E; show that ED, EF, EG are in harmonical proportion. (Euclid, vi. 4. Cape, iii. 69.)

86. If the four sides of a quadrilateral figure be bisected, the lines joining the points of bisection shall form a parallelogram, whose area equals half the area of the quadrilateral. (Euclid, vi. 2, 1. Cape, iii. 66.)

87. If two diagonals of a regular pentagon intersect; 1st, the greater segment is equal to a side of the pentagon; 2nd, the two diagonals cut each other in extreme and mean ratio. (Euclid, vi. 4. Cape, iii. 69.)

88. Within an isosceles triangle to find a point, such that its distance from one of the equal angles may equal twice its distance from the vertical angle. (Euclid, iii. 31, and vi. 4. Cape, ii. 47, and iii. 69.)

89. If two circles touch each other, and any two parallel diameters be drawn, the straight line joining their extremities towards the same or opposite parts, according as the circles touch internally or externally, shall pass through the point of contact. (Euclid, vi. 4, and v. 17, 18. Cape, iii. 69, and Proportion, art. 180.)

90. If two circles touch each other externally, and also a given straight line, the part of the line between the points of contact is a mean proportional between the diameters. (Euclid, vi. 17. Cape, Proportion, art. 186.)

91. If two circles touch each other, either internally or externally, any two straight lines drawn from the point of contact will be cut proportionally by the circumferences. (Euclid, vi. 4. Cape, iii. 69.)

92. If from one extremity of a chord a tangent be drawn to a circle, equal to the chord, and a line be drawn joining the further extremities of the chord and the tangent, the arc intercepted between that line and the tangent shall be equal to half the arc subtended by the chord. (Euclid, vi. 33.)

93. If a straight line, which touches two circles, cut another

straight line, which joins their centres, the segments of the latter will be proportional to the diameters. (Euclid, vi. 4. Cape, iii. 69.)

94. If from the extremities of any chord of a circle, perpendiculars be drawn to the chord, the points where they meet any diameter shall be equally distant from the centre. (Euclid, vi. 2. Cape, iii. 66.)

95. If a circle be inscribed in a triangle, and another circle be described touching the base and the other two sides produced; 1st, the points where the circles touch the base shall be equally distant from its extremities; 2nd, the distance between the points where they touch either one of the sides shall be equal to the base. (Euclid, vi. 4, and v. 22. Cape, iii. 69, and Proportion, art. 183.)

96. To describe a circle which shall pass through two given points and touch a given straight line. (Euclid, vi. 13. Cape, iii. 72.)

97. From a given point in the side of a triangle, to draw a straight line, which shall bisect the triangle. (Euclid, i. 37. Cape, iii. 53. Cor. 2.)

98. From a given angle of a trapezium, to draw a straight line, bisecting the trapezium. (Euclid, i. 37. Cape, iii. 53. Cor. 2.)

99. From a given point in the side of a triangle, to draw straight lines, which shall divide it into any number of equal parts. (Euclid, i. 37, and vi. 1. Cape, iii. 53. Cor. 2.)

100. To transform any rectilineal figure into a triangle, of equal area, whose vertex shall be in one of the angles of the figure, and its base in one of its sides. (Euclid, i. 37. Cape, iii. 53. Cor. 2.)

101. To transform any given triangle into an isosceles one of equal area. (Euclid, i. 37. Cape, iii. 53. Cor. 2.)

102. To transform any given isosceles triangle into an equilateral one of equal area. (Euclid, vi. 13. Cape, prob. 9.)

103. To divide a given straight line into two segments, such that the rectangle contained by them shall be a maximum. (Euclid, i. 10. Cape, prob. 1.)

104. Through a given point within a circle, to draw the least possible chord. (Euclid, iii. 7, 15. Cape, ii. 37.)

105. On a given base, to describe a triangle, having a given vertical angle, and whose area shall be a maximum. (Euclid, iii. 33, 15. Cape, prob. 22, and ii. 37.)

106. On a given base, to describe a triangle, another of whose sides is given, so that the area may be a maximum. (Euclid, i. 11. Cape, prob. 2.)

107. To divide a circle into any number of parts, which shall be equal both in area and in perimeter. (Euclid, v. 17, and xii. 2. Cape, iv. 87, and Proportion, art. 180.)

108. To divide a circle into any number of equal concentric an-

nuli. (Euclid, vi. 9, 8, xii. 2, and v. 17. Cape, iii. 72, iv. 87, and Proportion, art. 180.)

109. Given one side of a right-angled triangle, and the difference between the hypothenuse and the other side, to construct it. (Euclid, vi. 12, and i. 47. Cape, prob. 8, and iii. 61.)

110. Given the perpendicular from the right angle on the hypothenuse of a right-angled triangle, and the difference of the segments of the hypothenuse, to construct it. (Euclid, i. 47. Cape, iii. 61.)

111. Given the hypothenuse, and the sum of the two sides of a right-angled triangle, to construct it. (Euclid, i. 32, iii. 20, 21. Cape, i. 24, ii. 44, 45.)

112. Given the segments of the hypothenuse of a right-angled triangle, made by a perpendicular from the right angle, to construct it. (Euclid, iii. 31. Cape, ii. 47.)

113. Given the hypothenuse, and the difference of the sides of a right-angled triangle, to construct it. (Euclid, iii. 31, 26; i. 6. Cape, ii. 47; i. 9.)

114. Given the sides of a right-angled triangle in continued proportion, and the length of the hypothenuse, to construct it. (Euclid, ii. 11; vi. 8. Cape, prob. 10, and iii. 72.)

115. Given the base of a triangle, one of the angles at the base, and the difference of the two sides, to construct it. (Euclid, i. 6. Cape, i. 9.)

116. Given the base, the difference of the two angles at the base, and the difference of the two sides of a triangle, to construct it. (Euclid, i. 23, 6, 32. Cape, prob. 4, and i. 9, 24.)

117. Given the vertical angle of a triangle, and the segments into which the perpendicular from the vertex divides the base, to construct it. (Euclid, iii. 33. Cape, prob. 22.)

118. Given the base, the vertical angle, and the sum of the sides of a triangle, to construct it. (Euclid, iii. 33, 20. Cape, prob. 22, and ii. 44.)

119. Given the base, the ratio of the sides, and the vertical angle of a triangle, to construct it. (Euclid, iii. 33, vi. 10. Cape, prob. 22.)

120. Given the vertical angle of a triangle, the sum of its sides, and the difference of the segments into which a perpendicular from the vertex divides the base, to construct it. (Euclid, i. 23, 32; iii. 3. Cape, prob. 4; i. 24; ii. 35.)

MENSURATION.

AREAS OF PLANE FIGURES.

General Formulæ.

1. In a triangle, if a, b, c be the sides opposite to the angles A, B, C; d the perpendicular from A on a, p the semi-perimeter,

$$\text{area} = \frac{1}{2}ad = \frac{1}{2}bc\sin A = \sqrt{p(p-a)(p-b)(p-c)}.$$

2. In a parallelogram, if b, c be two sides including the angle A,

$$\text{area} = bc\sin A.$$

3. In a trapezoid, if a, b be the two parallel sides, d the perpendicular distance between them,

$$\text{area} = \frac{d}{2}(a+b).$$

4. In a regular polygon of n sides, if $2a$ be the length of one of the sides,

$$\text{area} = na^2\cot\frac{180^\circ}{n}.$$

5. In a circle, if $r =$ radius, $\pi = 3{\cdot}1416$,

circumference $= 2\pi r$,
area $= \pi r^2$.

6. In a circular ring, if a and b be the external and internal radii,

$$\text{area} = \pi(a^2-b^2).$$

7. In the sector of a circle, if n be the number of degrees at the centre,

length of arc : whole circumference of circle :: n : 360,
area of sector : area of circle :: n : 360.

8. In a parabola, if a be the height, and b the base,

$$\text{area} = \frac{2}{3}ab.$$

9. In an ellipse, if a and b be the semi-axes,

$$\text{area} = \pi ab.$$

1. Find the area of a square, whose side is 15 chains, 40 links.

2. Find the area of a rectangular field, whose dimensions are $50\frac{3}{4}$ yards, and 123 yards.

3. Find the area of a rhombus, whose side is 5 feet 7 inches, and perpendicular height 4 feet.

4. Find the area of a rhombus, whose side is 17 yards, and one of whose angles is $49^\circ\ 14'\ 15''$.

5. Find the area of the parallelogram, two adjacent sides of

which are 4 chains, and 5 chains 50 links respectively; the angle included between them being 16° 43′.

6. A triangular field 738 links long, and 583 links in the perpendicular, produces an income of £12 a-year. At how much an acre is it let?

7. How much ground is there in a triangular fish-pond, whose three sides measure 400, 348, and 312 yards respectively?

8. What is the area of a triangle, two adjacent sides of which are 24 and 17·6 yards, and include an angle of 30°?

9. Having a rectangular marble slab, 36 inches by 16 inches, I would have a square foot cut off it, parallel to the shorter side: I would then have the like quantity taken from the remainder, parallel to the longer side, and so on, as often as possible. What then would be the remainder?

10. I have to plant 10584 trees, at equal distances in rows, in a plantation whose length is six times its breadth. How many rows will there be in the shorter end?

11. If I place 3582 plants in rows, each 4 feet asunder, and the plants 7 feet apart; how much ground is taken up?

12. What is the area of the bottom of a bath, the sum of whose three equal sides is 125 feet?

13. How much paper $\frac{3}{4}$ yard wide will be required for a room, that is 22 feet long, by 14 feet wide, and 9 feet high; if there be 3 windows and 2 doors, each 6 feet by 3 feet?

14. How many square feet are there in a plank, whose length is 10 ft. 5 in., and the breadths of the two ends $2\frac{1}{2}$ feet, and $1\frac{3}{4}$ feet respectively?

15. How many square yards of paving are there in a quadrangular court, whose diagonal is 54 feet; and the perpendiculars on it from the opposite corners 25 and $17\frac{1}{4}$ feet?

16. Find the area of a regular pentagon, whose side is 15 feet.

17. Find the area of a regular hexagon, whose side is 15 feet.

18. Find the area of a regular heptagon, whose side is 15 feet.

19. Find the area of a regular octagon, whose side is 15 feet.

20. If the diameter of a circle be 5 feet, what is its circumference?

21. If the circumference of a circle be 10 chains, what is its radius?

22. What is the area of a circle, whose diameter is 12 feet?

23. How many square feet are there in a circle, whose circumference is 6·2832 feet?

24. What is the length of an arc of a circle containing $29\frac{1}{2}$°; the radius of the circle being 9 feet?

25. Find the area of the sector of a circle, whose radius is 55 yards; the length of the arc being 59 yards.

26. If the diameter of a circle be 84 inches, what is the area of a segment whose height is 30 inches?

27. What is the area of a segment of a circle, whose chord is 22 feet, and height 22 feet?

28. What is the area of the sector of a circle of radius 8 feet; if the arc of the sector contain 159°?

29. Find the area of a circular ring, whose diameters are 5 and 15 feet.

30. What is the area between two concentric semicircles, whose radii are 24 and 16 feet?

31. What is the area of the front of a circular arch, built with stones $3\frac{1}{2}$ feet long, whose upper and lower bounding arcs are respectively 84 and $72\frac{1}{2}$ feet in length?

32. How much will the turfing of a round plot cost at 4*d.* per square yard; if it be 130 feet round?

33. A curb 15 inches broad is put to a well 7 feet in diameter, and costs 17*s.* 6*d.*; at how much per square foot is that?

34. The extremity of the minute-hand of a dial moves 5 inches in $3\frac{1}{4}$ minutes; what is its length?

35. The paving of a semicircular alcove with marble at 2*s.* 6*d.* a foot came to £10; what was the length of the arc in front?

36. What is the area of a sector, whose radius is 30 yards, and arc 63 feet?

37. If the chord joining the two extremities of the above curve be 15 inches shorter than the arc; what is the area of the segment?

38. What is the area of an ellipse, whose axes are 34 and 30 feet?

39. The ellipse in Grosvenor-square measures 840 links the longer way, and 612 the shorter, within the rails; and the curb stones are 14 inches wide; what is the area of all the ground taken up?

40. A semicircular arch is made with stones 3 feet long; the span of the arc being 40 feet, and its height 16 feet; what is the area of the front of the arch?

41. Find the area of a parabola, whose height is 7 and base 12 yards.

SURFACES AND CONTENTS OF SOLIDS.

General Formulæ.

1. In a prism or cylinder, if p be the perimeter of the base, a^2 its area; h the height,

$$\text{surface} = ph + 2a^2$$
$$\text{volume} = a^2h.$$

2. In a pyramid or cone, if p be the perimeter of the base, a^2 its area; h the perpendicular height, and l the slant height,

$$\text{surface} = \frac{1}{2}pl$$

$$\text{volume} = \frac{1}{3} a^2 h.$$

3. In the frustum of a pyramid or cone, if p and q be the perimeters of the two ends; l the slant height; a^2 and b^2 the area of the two ends; h the perpendicular height,

$$\text{surface} = \frac{l}{2}(p+q),$$

$$\text{volume} = \frac{h}{3}(a^2+ab+b^2).$$

4. In a sphere whose radius is r,

$$\text{surface} = 4\pi r^2,$$

$$\text{volume} = \frac{4}{3}\pi r^3.$$

5. In the segment of a sphere, if h be its height, r the radius of the sphere,

$$\text{surface} = 2\pi hr,$$

$$\text{volume} = \frac{\pi h^2}{6}(6r-2h).$$

42. How many gallons of water will a cistern hold, whose length, breadth, and depth are 4 ft. 9 in., 3 ft. 6 in., and 2 ft. 9 in. respectively? (277·274 cubic inches = one gallon.)

43. What is the content of a cylinder whose diameter is $4\frac{1}{2}$ feet, and height 8 feet?

44. Find the solidity and superficies of a square pyramid, each side of the base being 12 feet; and the perpendicular drawn on one of the triangular faces from the vertex to the base being 25 feet.

45. Find the solid content and superficies of a triangular pyramid, each side of whose base is $5\frac{1}{2}$ feet, and the perpendicular height 30 feet.

46. Find the convex surface of a cone, the diameter of whose base is $3\frac{1}{2}$ feet, and altitude 6 feet.

47. How many cubic feet are there in the conic frustum, the circumferences of whose ends are 66 and 56 feet respectively, and altitude 4 feet?

48. Find the surface and solid content of the frustum of a square pyramid, whose perpendicular height is 10 feet; each side of the greater end being 3 ft. 4 in., and each side of the less 2 ft. 2 in.

49. Find the volume of the frustum of an octagonal pyramid, whose perpendicular height is 6 feet; and each side of the two ends 4 feet and 5 feet respectively.

50. If a segment of 6 feet slant height be cut off a cone whose slant height is 30 feet, and the circumference of the base 10 feet; what is the surface of the frustum remaining?

51. Find the surface and solid content of an hexagonal pyramid, whose height is 10 feet, and each side of the base 2 ft. 6 in.

52. What is the solidity of the wedge, whose base measures 30 feet by 16 feet, and whose height is 12 feet?

53. How many cubic feet of water can be contained in the ditch that measures 304 feet by 20 at the top, and 300 feet by 16 at the bottom; the uniform depth being 5 feet?

54. Find the surface and the solid content of a globe whose diameter is 9 feet.

55. Find the surface and the solid content of the segment of a sphere, 10 feet in height; the radius of the sphere being 20 feet.

56. Find the convex surface of a slice 2 feet high, cut from a globe of 17 feet radius.

57. How much of the earth's surface would a man see, if he were raised to the height of the radius above it? Find also what portion of the volume is contained in the segment of which he sees the surface.

58. To what height must a man be raised above the earth, in order that he may see the sixth part of its surface? Find also in this case what portion of the volume is contained in the segment.

59. The radius of the earth being 4000 miles, if geologists know the interior to the depth of 5 miles beneath the surface, what portion of the globe's solid content is known?

60. If the diameter of the earth be 8000 miles, and the height of the atmosphere 45 miles, what is the solid content of the latter?

61. The diameter of the earth being as before, if the pressure of the atmosphere at the surface be 15 lbs. to a square inch, how much weight of atmosphere does the earth support?

62. What is the difference between a solid half foot, and half a solid foot?

63. What is the ratio between the space of a room 25 feet 6 inches long, 20 feet 2 inches broad, and 14 feet high; and the space of two other equal rooms of half those dimensions?

64. A ship's hold is 102 feet long, 40 feet broad, and 5 feet deep; how many bales of goods 3 feet 6 inches long, 2 feet 3 inches broad, and 2 feet 6 inches deep, can be stowed into it, leaving a gang-way of 4 feet broad?

65. A rectangular cistern 8 feet long, and 4 feet broad, is made of lead, every square foot of which weighs 16 lbs.; while the whole cistern weighs a ton. How many gallons of water will it hold?

66. A log of timber is 18 feet long, 18 inches broad, and 14 inches thick. If $2\frac{1}{2}$ solid feet be cut off the end of it, what length will be left?

67. A stone 20 inches long, 15 broad, and 8 deep, weighs 280 lbs. How many cubic feet of this kind of stone will freight a vessel of 240 tons burden?

The standard thickness of brick-work is 3 half bricks; any other thickness must be reduced to the standard by multiplying by the number of half bricks, and dividing by 3.

The standard rod of brick-work contains $272\frac{1}{4}$ square feet.

68. How many standard rods of brick-work are there in a wall 60 feet long, 12 feet high, and 3 bricks thick?

69. A triangular gable 18 feet high, of 1 brick thick, is raised on an end wall, 20 feet long, and 30 feet high, of 2 bricks thick. What is the cost of the whole at £2 a standard rod?

70. A marble chimney-piece is to be put up for 10*s.* a square foot. If the length of the slab be 4 feet 6 inches, the breadth of the mantel and slab together 1 foot, the height of each jamb 4 feet 3 inches, and the breadth of each 1 foot; what is the whole cost?

71. What does the wainscoting of a room cost, at £3 15*s.* a square of 100 feet, if the length, breadth, and height be respectively 21, 15 and 10 feet; the door, which measures 6 feet by 4, and two window shutters, each 5 feet by 4, being reckoned work and half work?

72. What is the expense of tiling a house at 25*s.* a square; the length and breadth of the house being 50 feet and 30 feet respectively; the girt over being $\frac{3}{2}$ of the breadth of the house; and both top and bottom of the eaves, which project 1 foot on each side, being counted into the work?

73. What is the expense of a leaden pipe of 2 inches bore, half an inch thick, and 4 yards long, at $2\frac{1}{2}$*d.* a pound; the cubic foot weighing 11325 ounces?

MEASUREMENT OF SHOT, SHELLS AND POWDER.

In the following questions it is assumed that,—

1. An iron ball of 4 inches diameter weighs 9 lbs.
2. A leaden ball of 1 inch diameter weighs $\frac{3}{14}$ lb.
3. 30 cubic inches of gunpowder weigh 1 lb.

74. What is the weight of an iron ball of 1 foot diameter?

75. What is the diameter of an iron ball which weighs 200 lbs.?

76. What is the weight of an iron shell, the external and internal diameters of which are 9 inches and 6 inches respectively?

77. If the outer diameter of an iron shell that weighs 75 lbs. be 9 inches; what is the inner diameter?

78. What is the weight of a leaden ball of 3 inches diameter?

79. What is the diameter of a leaden ball which weighs 4 lbs?

80. What weight of powder will fill a box, whose dimensions are 2 feet, $1\frac{1}{2}$ foot, and 1 foot respectively?

81. How many inches are there in each side of a cubical box that holds 100 lbs. of powder?

82. How much powder will fill a shell, whose internal diameter is 9 inches?

83. What is the internal diameter of a shell that holds 10 lbs. of powder?

84. What weight of powder will fill a cylinder, whose height is 3 feet, and diameter of base 10 inches?

85. What length of a cylinder of 6 inches diameter will be filled with 10 lbs. of powder?

86. What is the radius of the base of a cylinder 4 feet high, that holds 100 lbs. of powder?

87. What weight of powder will fill a cone, whose altitude is 7 feet, and diameter of base 2 feet?

88. What is the altitude of a cone, the radius of whose base is 9 inches, that will hold 50 lbs. of powder?

89. What is the radius of the base of a cone, whose altitude is 8 feet, and which holds 450 lbs. of powder?

TRIGONOMETRY.

TRIGONOMETRICAL FORMULÆ.

1. Express in degrees and in grades, two angles, whose sum is 75 degrees, and difference 25 grades.

2. Find the ratios of the lengths of the decimal and the sexagesimal degree, minute, and second, respectively.

3. Find the circular measure of the angles 57°, $57^\circ\ 30'$, 18°, 54°, and $22^\circ\ 30'$.

4. Find the angles whose circular measures are 1, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{5}{6}$.

Adapt to radius r, the following formulæ:—

5. $\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}$.

6. $\sin 3A=3\sin A-4\sin^3 A$.

7. $\cot A=\frac{\sin A}{\cos A\,.\,\tan^2 A}$.

8. $\tan 3A.\tan A=\frac{\sin 3A.\sin A}{\cos 3A.\cos A}$.

9. $\tan^2 2A-\tan^2 A=\frac{\sin 3A.\sin A}{\cos^2 2A.\cos^2 A}$.

10. $1+\cos 2A.\cos 2B=\cos^2(A+B)+\cos^2(A-B)$.

11. If l be the length of an arc of 45° to radius r, l' the length of an arc of 30° to radius r', show that $\frac{2l}{3l'}=\frac{r}{r'}$.

Prove the following formulæ:—

12. $\frac{\cos B-\cos A}{\cos B+\cos A}=\tan\frac{1}{2}(A+B)\,.\,\tan\frac{1}{2}(A-B)$.

13. $\cot^2 A\times\cos^2 A=\cot^2 A-\cos^2 A$.

14. $\operatorname{cosec} 2A=\frac{1+\cot^2 A}{2\cot A}$.

15. $\sec^2 A\times\operatorname{cosec}^2 A=\sec^2 A+\operatorname{cosec}^2 A$.

16. $\cos A=\cos^4\frac{A}{2}-\sin^4\frac{A}{2}$.

17. $\frac{\cos^2 A-\sin^2 B}{\sin^2 A.\sin^2 B}=\cot^2 A.\cot^2 B-1$.

18. $\frac{\tan A+\tan B}{\cot A+\cot B}=\tan A\,.\,\tan B$.

19. $\tan A = \dfrac{2 + 2\tan A \,.\, \tan\dfrac{A}{2}}{\tan\dfrac{A}{2} + \cot\dfrac{A}{2}} = (1 + \sec A) \,.\, \tan\dfrac{A}{2}.$

20. $\tan 2A - \tan A = \dfrac{2 \sin A}{\cos A + \cos 3A}.$

21. $\dfrac{\sin 2A}{1 + \cos 2A} \times \dfrac{\cos A}{1 + \cos A} = \tan\dfrac{A}{2}.$

22. $\dfrac{2 \sin A + \sin 2A}{2 \sin A - \sin 2A} = \cot^2\dfrac{A}{2}.$

23. $\sin 5A \,.\, \sin A = \sin^2 3A - \sin^2 2A.$

24. $\tan 3A \,.\, \tan A = \dfrac{\cos 2A - \cos 4A}{\cos 2A + \cos 4A}.$

25. $\dfrac{1 - 2 \sin^2 A}{1 + \sin 2A} = \dfrac{1}{\sec 2A + \tan 2A}.$

26. $\tan^2 2A - \tan^2 A = \dfrac{\sin 3A \,.\, \sin A}{\cos^2 2A \,.\, \cos^2 A}.$

27. $\cos A\{1 - \tan 2A \,.\, \tan A\} = \cos 3A\{1 + \tan 2A \,.\, \tan A\}.$

28. $\sin A = \dfrac{\left\{1 + \tan\dfrac{A}{2} + \sec\dfrac{A}{2}\right\}\left\{1 + \tan\dfrac{A}{2} - \sec\dfrac{A}{2}\right\}}{\sec^2\dfrac{A}{2}}.$

29. $\tan^4 A = \dfrac{\sin^2 2A - 4 \sin^2 A}{\sin^2 2A + 4 \sin^2 A - 4} = \dfrac{\cos^2 2A - 4 \cos^2 A + 3}{\cos^2 2A + 4 \cos^2 A - 1}.$

30. $\tan A = \dfrac{\sin\dfrac{2A}{3} + \sin\dfrac{4A}{3}}{\cos\dfrac{2A}{3} + \cos\dfrac{4A}{3}}.$

31. $\tan 2A = \dfrac{\sin A + \sin 3A}{\cos A + \cos 3A}.$

32. $\tan 3A = \dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}.$

33. $\sin A + \sin 2A + \sin 3A = \sin 2A\left\{3 - 4 \sin^2\dfrac{A}{2}\right\}.$

34. $1 + \cos A + \cos 2A + \cos 3A = 2 \cos A\{2 \cos^2 A + \cos A - 1\}.$

35. $\{\cos A + \sin A \sqrt{-1} - 1\}\{\cos A - \sin A \sqrt{-1} - 1\} = 4 \sin^2\dfrac{A}{2}.$

36. $\{2 \sin A + \sin 2A\}\tan^2\dfrac{A}{2} = 2 \sin A - \sin 2A.$

37. $\left\{1 - \tan\dfrac{A}{2}\right\}\tan A = 1 - \sec A + \tan\dfrac{A}{2}(1 + \sec A).$

38. $\tan 3A \,.\, \tan A = \dfrac{\tan^2 2A - \tan^2 A}{1 - \tan^2 A \,.\, \tan^2 2A}$.

39. $\text{chd}\,\dfrac{3A}{2} \cdot \text{chd}\,\dfrac{A}{2} = \text{chd}^2 A - \text{chd}^2 \dfrac{A}{2}$.

40. $1 + \cos 2A \,.\, \cos 2B = \cos^2(A+B) + \cos^2(A-B)$.

41. $\sec(A \pm B) = \dfrac{\sec A \,.\, \sec B \,.\, \text{cosec}\, A \,.\, \text{cosec}\, B}{\text{cosec}\, A \,.\, \text{cosec}\, B \mp \sec A \,.\, \sec B}$.

42. $\text{vers}(A-B) \,.\, \text{vers}\{\pi - (A+B)\} = \sin A - \sin B)^2$.

43. $\cos A + \cos(A + 2B) = 2\left(2\cos^2 \dfrac{B}{2} - 1\right) . \cos(A+B)$.

44. $\sin(A+B) \,.\, \sin 3(A-B) = \sin^2(2A-B) - \sin^2(2B-A)$.

45. $\cos(A-B-C) + \cos(A-B+C) + \cos(A+B-C)$
$+ \cos(A+B+C) = 4 \cos A \cos B \cos C$.

46. $\sin(A-B) \,.\, \sin C - \sin(A-C) . \sin B + \sin(B-C) . \sin A = 0$.

47. $\sin(A+B) \,.\, \sin(B+C+D)$
$= \sin A \,.\, \sin(C+D) + \sin B \,.\, \sin(A+B+C+D)$.

48. $\cos A + \cos(A+2B) = 2 \cos B \,.\, \cos(A+B)$.

49. If S and s be the sines of two arcs, C and c their cosines,
$(S^2 - S^2 s^2)(C^2 - C^2 c^2) = (s^2 - s^2 S^2)(c^2 - c^2 C^2)$.

50. If T and t be the tangents, S and s the secants of two arcs,
$$\frac{S^2(s^2-1)}{s^2(S^2-1)} = \frac{t^2(T^2+1)}{T^2(t^2+1)}.$$

---

51. Find the values of the sines of the following angles,
7° 30′, 9°, and 22° 30′.

52. Find the cosines of the following angles,
12°, 11° 15′, and 33° 45′.

53. Find the tangents of the following angles,
18°, 22° 30′, 37° 30′.

54. Find the cotangents of the following angles,
36°, 75°, 225°.

55. Find the secants of the following angles,
22° 30′, 54°, 225°.

56. Find the cosecants of the following angles,
30°, 72°, 120°.

57. Find the versines of the following angles,
15°, 67° 30′, 240°.

58. Find the chords of the following angles,
36°, 45°, 240°.

Prove the following:

59. $\sin 3A = 4 \sin A \,.\, \sin(60° + A) \,.\, \sin(60° - A)$.

60. $2 \sec 2A = \sec(45° + A) \,.\, \sec(45° - A)$.

61. $\tan\left(45° + \dfrac{A}{2}\right) + \cot\left(45° + \dfrac{A}{2}\right) = 2 \sec A$.

62. $\tan(30^\circ+A)\,.\,\tan(30^\circ-A)=\dfrac{2\cos 2A-1}{2\cos 2A+1}$.

63. $\tan 50^\circ+\cot 50^\circ=2\sec 10^\circ$.

64. $\dfrac{\sin 45^\circ-\sin 30^\circ}{\sin 45^\circ+\sin 30^\circ}=\dfrac{\sec 45^\circ-\tan 45^\circ}{\sec 45^\circ+\tan 45^\circ}$.

65. $\dfrac{\sin 60^\circ-\sin 30^\circ}{\sin 60^\circ+\sin 30^\circ}=\dfrac{\tan 60^\circ-\tan 45^\circ}{\tan 60^\circ+\tan 45^\circ}$.

66. $\tan\left(45^\circ+\dfrac{A}{2}\right)=\sec A+\tan A$.

67. $2\,\text{vers}\left(90^\circ+\dfrac{A}{2}\right).\,\text{vers}\left(90^\circ-\dfrac{A}{2}\right)=\text{vers}\,(180^\circ-A)$.

68. $\text{chd}^2(90^\circ-A)=2-2\sin A$.

Prove the following formulæ, where $A+B+C=90^\circ$.

69. $\tan A\,.\,\tan B+\tan A\,.\,\tan C+\tan B\,.\,\tan C=1$.

70. $\cot A+\cot B+\cot C=\cot A\,.\,\cot B\,.\,\cot C$.

71. $\tan A+\tan B+\tan C=\tan A.\tan B.\tan C+\sec A.\sec B.\sec C$.

72. $\sin 2A+\sin 2B+\sin 2C=4\cos A\,.\,\cos B\,.\,\cos C$.

Prove the following formulæ, where $A+B+C=180^\circ$.

73. $\sin 2A+\sin 2B+\sin 2C=4\sin A\,.\,\sin B\,.\,\sin C$.

74. $\sin^2\dfrac{A}{2}+\sin^2\dfrac{B}{2}+\sin^2\dfrac{C}{2}+2\sin\dfrac{A}{2}\cdot\sin\dfrac{B}{2}\cdot\sin\dfrac{C}{2}=1$.

75. $\cos^2 A+\cos^2 B+\cos^2 C+2\cos A\,.\,\cos B\,.\,\cos C=1$.

76. $4\cos\dfrac{A}{2}\cdot\cos\dfrac{B}{2}\cdot\cos\dfrac{C}{2}=\sin A+\sin B+\sin C$.

77. $\sin(A+B)\,.\,\sin(B+C)=\sin A\,.\,\sin C$.

78. $\tan A+\tan B+\tan C=\tan A\,.\,\tan B\,.\,\tan C$.

79. $\cot A\,.\,\cot B+\cot A\,.\,\cot C+\cot B\,.\,\cot C=1$.

80. If $A+B+C=45^\circ$, prove that
$\tan A+\tan B+\tan C-\tan A\,.\,\tan B\,.\,\tan C$
$=1-\tan A\,.\,\tan B\,.-\tan A\,.\,\tan C-\tan\,.\,\tan C$.

If A, B and C be in arithmetical progression, prove the following formulæ:—

81. $\sin A-\sin C=2\sin(A-B)\,.\cos B$.

82. $\dfrac{\tan B}{\tan(B-C)}=\dfrac{\sin A+\sin C}{\sin A-\sin C}=\dfrac{\tan\dfrac{A+C}{2}}{\tan\dfrac{A-C}{2}}$.

82.*$(\sin A-\sin C)\,.\,\sin B=(\cos C-\cos A)\,.\,\cos B$.

83. $\dfrac{1}{\tan A+\tan C}-\dfrac{1}{2\tan B}=\dfrac{1}{\cot A+\cot C}-\dfrac{1}{2\cot B}$.

---

84. If $\cos(A-C)\,.\,\cos B=\cos(A-B+C)$, prove that the tangents of A, B and C are in harmonical progression.

85. If $1 = \left(\frac{\sin A}{\sin B}\right)^2 + (\cos A \,.\, \cos C)^2$, then $\sin C = \frac{\tan A}{\tan B}$.

86. If $\tan A = \frac{b \sin B}{a + b \cos B}$, then $\tan (B - A) = \frac{a \sin B}{b + a \cos B}$.

87. If $\sin B = m \sin (2A + B)$, then $\tan (A + B) = \frac{1 + m}{1 - m} \tan A$.

88. If $\cos A = \cos B \,.\, \cos C$, then $\tan \frac{A+B}{2} \cdot \tan \frac{A-B}{2} = \tan^2 \frac{C}{2}$.

89. If $\sec A \,.\, \sec B + \tan A \,.\, \tan B = \sec C$,
then $\tan B = \pm \frac{\sin C \mp \sin A}{\cos C \,.\, \cos A}$.

Determine A from the following equations :—

90. $\sin A = \sin 2A$. 91. $\tan 2A = 3 \tan A$.
92. $\tan A = \operatorname{cosec} 2A$. 93. $\cos A = \tan A$.
94. $\tan A + 3 \cot A = 4$. 95. $2 \sin^2 3A + \sin^2 6A = 2$.
96. $\cos nA + \cos (n-2)A = \cos A$.
97. $\tan^2 A + 4 \sin^2 A = 6$. 98. $2 \sin A = \tan A$.
99. $\tan A + \cot A = 4$. 100. $m \sin A = n \cos^2 A$.
101. $m = \sin A . \cos A$. 102. $\cos 3A + \cos 2A + \cos A = 0$.
103. $4 \sin A . \sin 3A = 1$.

Determine x from the following equations :—

104. $\sin (x + \alpha) + \cos (x + \alpha) = \sin (x - \alpha) + \cos (x - \alpha)$.

105. $\tan (x + \alpha) . \tan (x - \alpha) = \frac{1 - \cos 2x}{1 + \cos 2x}$.

106. $\tan \alpha \,.\, \tan x \,. = \tan^2 (\alpha + x) - \tan^2 (\alpha - x)$.

107. $\{ \sqrt{1 + \sin x} - 1 \} . \{ \sqrt{1 - \sin x} + 1 \} = \tan \frac{\alpha}{2} \cdot \sin x$.

Prove the following :—

108. $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = 45°$.

109. $\tan^{-1} \frac{1}{7} + 2 \tan^{-1} \frac{1}{3} = 45°$.

110. $\cot^{-1} \frac{3}{4} + \cot^{-1} \frac{1}{7} = 135°$. 111. $\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 = 45°$.

112. $\cos^{-1} \sqrt{\frac{2}{3}} - \cos^{-1} \left(\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{3}} \right) = 30°$.

113. If $\theta = \tan^{-1} \frac{1}{\sqrt{3}}$, and $\phi = \tan^{-1} \frac{1}{\sqrt{15}}$, then
$\sin (\phi + \theta) = \sin 60° \,.\, \cos 36°$.

114. Find x from the equation $\operatorname{vers}^{-1} (1 + x) - \operatorname{vers}^{-1} (1 - x)$
$= \tan^{-1} 2 \sqrt{1 - x^2}$.

115. If $\cos v = \frac{\cos u - e}{1 - e \cos u}$, find the value of $\tan \frac{v}{2}$ in terms of $\tan \frac{u}{2}$.

116. If $\tan\theta=\dfrac{\sin\phi\,.\,\cos\psi}{\sin\psi+\cos\theta}$, find the value of $\tan\dfrac{\theta}{2}$ in terms of $\tan\dfrac{\phi}{2}$ and $\tan\dfrac{\psi}{2}$.

117. If $\tan 2\theta=\dfrac{b}{a}$, find the value of $a\cos 2\theta+b\sin 2\theta$.

118. If $\tan\theta=\left(\dfrac{b}{a}\right)^{\frac{1}{3}}$, find the value of $\dfrac{a}{\cos\theta}+\dfrac{b}{\sin\theta}$.

119. If $\sin\theta=\sin\alpha\,.\,\sin(\phi+\theta)$, find $\tan\theta$ in terms of α and ϕ.

120. If $\tan\theta=\cos\alpha\,.\,\tan\phi$, find $\tan(\phi-\theta)$ in terms of α and ϕ.

121. If $\tan\theta=\tan^3\dfrac{\phi}{2}$, and $\cos^2\phi=\dfrac{m^2-1}{3}$, find m in terms of θ.

122. Eliminate θ from the equations

$$m=\operatorname{cosec}\theta-\sin\theta,\text{ and } n=\sec\theta-\cos\theta.$$

123. Eliminate θ and ϕ from the equations

$$\cos^2\theta=\frac{\cos\alpha}{\cos\beta},\ \cos^2\phi=\frac{\cos\gamma}{\cos\beta},\text{ and }\frac{\tan\theta}{\tan\phi}=\frac{\tan\alpha}{\tan\gamma}.$$

124. Eliminate θ and ϕ from the equations

$$a\sin^2\theta+b\cos^2\theta=m,$$
$$b\sin^2\phi+a\cos^2\phi=n,$$
$$\text{and }\ a\tan\theta=b\tan\phi.$$

PROPERTIES OF PLANE FIGURES.

125. In any triangle, any one side is equal to the sum of the products of each of the other sides into the cosine of the adjacent angle.

In any triangle, where a, b, c are the sides respectively opposite to the angles A, B, C, prove the following formulæ:—

126. $\sin(A-B)=\dfrac{a^2-b^2}{c^2}\cdot\sin(A+B)$.

127. $\sin\dfrac{A-B}{2}=\dfrac{a-b}{c}\cdot\cos\dfrac{C}{2}$.

128. $\sin^2\dfrac{A}{2} : \sin^2\dfrac{B}{2}=\dfrac{a}{b} : \dfrac{b+c-a}{a+c-b}$.

129. $\cos A+\cos B=\dfrac{a+b}{c}\times 2\sin^2\dfrac{C}{2}$.

130. $\tan B=\dfrac{b\,.\sin C}{a-b\,.\cos C}$.

131. $\dfrac{\tan B}{\tan C}=\dfrac{a^2+b^2-c^2}{a^2+c^2-b^2}$.

132. $\tan\frac{A}{2}\cdot\tan\frac{B}{2}=\frac{a+b-c}{a+b+c}$.

133. $$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{\tan\frac{A}{2}-\tan\frac{B}{2}}=\frac{c}{a-b}.$$

134. $\cos\frac{A-B}{2}=\frac{a+b}{c}\cdot\sin\frac{C}{2}$.

135. $\frac{a^2+b^2+c^2}{2}=ab\,.\cos C+ac\,.\cos B+bc\,.\cos A$.

136. $2a\,.\cot\frac{A}{2}=(b+c-a).\left(\cot\frac{B}{2}+\cot\frac{C}{2}\right)$.

137. $\frac{\operatorname{vers} A}{\operatorname{vers} B}=\frac{a(a+c-b)}{b(b+c-a)}$.

In any right-angled triangle, C being the right angle, and c the hypothenuse, prove the following formulæ:—

138. $\cos(A-B)=\frac{2ab}{c^2}$. 139. $\cos 2A=\frac{b^2-a^2}{b^2+a^2}$.

140. $\cos(45^\circ\pm A)=\frac{1}{\sqrt{2}}\cdot\left(\frac{b\pm a}{c}\right)$.

141. $\cos(2A-B)=\frac{a}{c^3}(3c^2-4a^2)$.

142. $\tan 2A=\frac{2ab}{b^2-a^2}$. 143. $\tan\frac{A}{2}=\sqrt{\frac{c-b}{c+b}}$.

144. In any triangle, the area

$$=\frac{ab}{2}\sin C=\frac{c^2}{2}\cdot\frac{\sin A\,.\sin B}{\sin C}$$

$$=\frac{2abc}{a+b+c}\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cos\frac{C}{2}=\frac{a^2-b^2}{2}\cdot\frac{\sin A\,.\sin B}{\sin(A-B)}$$

$$=\frac{(a+b+c)^2}{4}\cdot\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}.$$

145. In any right-angled triangle, C being the right angle, the

$$\text{area}=\frac{a}{2}\sqrt{c^2-a^2}=\frac{(a+b+c)(a+b-c)}{4}.$$

$$=\frac{c^2}{4}\sin 2A=\frac{b^2}{2}\tan A.$$

146. If R be the radius of the circle circumscribed about any triangle,

$$R=\frac{a+b+c}{8\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}}=\frac{1}{2}\sqrt[3]{\frac{abc}{\sin A\,.\sin B\,.\sin C}}.$$

147. If r be the radius of the circle inscribed in any triangle,

$$r=\frac{a+b+c}{2}\cdot\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}.$$

148. Using the same notation, $2R.r=\dfrac{abc}{a+b+c}$.

149. If R be the radius of the circle circumscribed about a right-angled triangle, r the radius of the inscribed circle,

$$R+r=\frac{a+b}{2}.$$

150. If a line CE bisect the angle C of any triangle, prove that

$$\tan AEC=\frac{a+b}{a-b}\cdot\tan\frac{C}{2},$$

$$\text{and } CE=\frac{2ab.\cos\frac{C}{2}}{a+b}=\frac{\sqrt{ab}}{a+b}\left\{(a+b)^2-c^2\right\}^{\frac{1}{2}}.$$

151. If CD bisect the side AB of any triangle,

$$CD^2=\frac{2a^2+2b^2-c^2}{4}.$$

152. In any triangle, if a perpendicular be drawn from C on AB, the distance of this perpendicular from the middle point of AB

$$=\frac{c}{2}\cdot\frac{\tan A-\tan B}{\tan A+\tan B}.$$

153. The sum of the perpendiculars from the angles of any triangle on the opposite sides $=2\text{ area}\left(\dfrac{ab+ac+bc}{abc}\right)$.

154. If through a point O within a triangle, three straight lines be drawn from the angles A, B, and C, meeting the opposite sides in D, E, F, then shall

$$\frac{OD}{AD}+\frac{OE}{BE}+\frac{OF}{CF}=1.$$

155. If $2\cos B=\dfrac{\sin A}{\sin C}$, the triangle is isosceles.

156. If $\dfrac{\tan A}{\tan B}=\dfrac{\sin^2 A}{\sin^2 B}$, the triangle is either isosceles or right-angled.

157. In an isosceles triangle, the versine of the vertical angle

$$=\frac{\text{square of base}}{\text{2 square of either side}}.$$

158. In any triangle, the sines of the angles are inversely as the perpendiculars let fall from them on the opposite sides.

159. The sides of a plane triangle are 3, 5, and 6; find the radii of the inscribed and circumscribed circles.

160. Find the area of a triangle, whose sides are 24, 30, and 18.

161. Find the area of a triangle, whose sides are 13, 12, and 5.

162. Given the ratios of the sines of the angles of a triangle $(1:m:n)$, and the radius of the inscribed circle (r); construct it.

163. In a triangle, having given B, a, and the area of the triangle, to construct it.

164. Find expressions for the three sides of a triangle, having given the radius of the circumscribing circle, and the three angles.

165. The sides of a triangle are in arithmetic progression, and its area is to the area of an equilateral triangle of the same area as $3:5$; find the ratio of the sides, and the value of the largest angle.

166. The angles of a plane triangle form a geometric progression whose common ratio is $\frac{1}{2}$, show that the greatest side $=2$ (perimeter) $\sin 12^\circ\ 51'\ 25\frac{5}{7}''$.

167. If the side of a pentagon inscribed in a circle be 1, find the value of the radius of this circle.

168. Compare the areas of regular pentagons and octagons described in and about a circle.

169. Find the value of the side of a regular figure of 24 sides in terms of the radius of the circumscribed circle.

170. Find the ratio between the sides of an equilateral triangle and a square inscribed in the same circle.

171. Find the ratio between the areas of an equilateral triangle and a square inscribed in the same circle.

172. The square of the side of a regular pentagon inscribed in a circle is equal to the sum of the squares of the sides of a regular hexagon and a decagon inscribed in the same circle.

173. In a right-angled triangle, if a perpendicular be drawn from the right angle to the hypothenuse, then the areas of the two circles inscribed in the triangles on each side of this perpendicular are proportional to the corresponding segments of the hypothenuse.

174. In the figure of Euclid, Book iv., prop. 10; compare the areas of the two circles.

175. The area of a regular polygon inscribed in a circle is a mean proportional between the areas of an inscribed and a circumscribed regular polygon of half the number of sides.

176. In a regular polygon of n sides, if each side $=2a$; find the sum of the radii of the circumscribed and inscribed circles.

177. The alternate angles of a regular pentagon are joined by lines, which form another pentagon; the alternate angles of this pentagon are joined, and so on. Given a side (a) of the first pentagon; find the sum of the areas of all the pentagons continued *ad infinitum.*

178. A circle is inscribed in an equilateral triangle; an equilateral triangle in this circle; and again a circle in this triangle, and so on, *ad infinitum.* Show that the radius of the first circle is equal to the sum of the radii of all the other circles.

179. If r_0 denote the radius of the circle inscribed in any triangle; r_1, r_2, r_3 the radii which touch each side respectively, and the other

two produced; find r_0 in terms of r_1, r_2, r_3, and S the area of the triangle in terms of r_0, r_1, r_2, r_3.

180. The distance between the centres of two wheels is a, and the sum of their radii c; find the whole length of a string which crosses between them, and just wraps round them.

181. Find an expression for the sum of the straight lines drawn from any one of the angular points of a regular polygon of n sides (each side of which $=2a$) to all the other angular points.

182. In building a honey-comb the bee employs less wax by making the cells of an hexagonal shape, than of the shape of any other regular figure of equal area.

> It is necessary to show what figures only can be used; and to find the ratios between the sums of the lines of division for each figure: considering the whole area taken up as infinitely great in comparison of the area of one of the cells.

SOLUTION OF PLANE TRIANGLES.

Find the remaining parts of the triangle ABC, where the following parts are given.

183. C =90°.
A =18° 14′.
AB=432.

184. C =90°.
A =42° 35′.
BC=296·35.

185. C =90°.
B =25° 19′ 13″.
AB=1234.

186. C =90°.
B =56° 53′ 35″.
AC=184.

187. C =90°.
AB=957·34.
AC=24.

188. C =90°.
AB=3562.
BC=357.

189. C =90°.
AC=423.
BC=584.

190. A =31° 13′.
B =48° 24′ 15″.
BC=926·7.

191. B =39° 15′.
C =13°.
AC=326.

192. AB=527.
AC=635·91.
B =88° 14′.

193. AC=333.
BC=325.
A =52° 19′.

194. AB=1249·6.
BC= 397·3.
A =8° 19′ 35″.

195. C =124° 34′.
AC=126.
AB=264.

196. A =53° 14′.
AB=521·05.
BC=417·4026.

197. AB=426.
AC=354.
A =49° 16′.

198. AB= 53·94.
BC=156·5.
B =15° 13′ 14″.

199. AB = 1263.
AC = 1359.
BC = 1468.

200. AB = 12·6356.
AC = 13·4837.
BC = 11·2983.

HEIGHTS AND DISTANCES.

201. At 120 feet distance from the foot of a steeple, the angle of elevation of the top was found to be 60° 30′. Required the height.

202. From the top of a rock 326 feet above the sea, the angle of depression of a ship's bottom was found to be 24°. Required the distance of the ship.

203. A wall is surrounded by a ditch; from the edge of this ditch the angle of elevation of a point on the top of the wall is found to be 35°; and at a distance of 100 yards from the ditch the angle of elevation of the same point is found to be 15°. Find the height of the wall; the breadth of the ditch; and the length of the ladder that would just reach from the edge of the ditch to the top of the wall.

204. From the top of a hill I observe two milestones in a straight line before me; and find their angles of depression to be respectively 5° and 15°. Find the height of the hill.

205. Two observers, on the same side of a balloon, and in the same vertical plane with it, a mile apart, respectively find its angles of elevation to be 15° and 65° 30′ at the same instant. Find the height of the balloon.

206. A ladder 38 feet long, just reaches to a window 29 ft. 6 in. high on one side of a street; and, on turning the ladder over without moving its foot it reaches a window 28 feet high on the other side. Find the breadth of the street.

207. The top of a maypole being broken off, struck the ground at a distance of $13\frac{3}{8}$ feet from the bottom of the pole; and the broken piece was found to measure $29\frac{4}{5}$ feet. Find the original height of the pole.

208. The aspect of a wall 18 feet high is due south, and the length of the shadow cast on the north side at noon is 16 feet. Find the sun's altitude, or the angle of elevation of the sun above the horizon.

209. At a distance of 200 yards from the foot of a church tower, the angle of elevation of the top of the tower was 30°, and of the top of the spire on the tower 32°. Find the height of the tower and of the spire.

210. Two men are surveying: when each is at a distance of 200 yards from the flag-staff, the one finds the angle subtended by the position of his companion and the staff to be 30° 15′. Find how far they are apart.

211. In order to ascertain the height of a castle on the top of a cliff, I measured 240 yards directly from it, and at the two ends of this line, found the angles of elevation of the top of the castle to be 29° and 13° 16′ respectively; also at the further end of the line, the castle's height subtended an angle of 5° 15′. Find the height.

212. Wishing to know the breadth of a moat surrounding a fortress, I measured a ground line of 30 yards by its side, and at the two ends found the angles subtended by the other end and a corner of the wall close by the other side of the moat to be 32° 19′ and 37° 46′ respectively, while the angle of elevation at the first end was 58° 32′. Find the breadth of the moat.

213. Two ships, half a mile apart, find that the angles subtended by the other ship and a fort, are respectively 56° 19′ and 63° 41′. Find the distance of each ship from the fort.

214. From the top of a house, 42 feet high, I found the angle of elevation of the top of a neighbouring steeple to be 14° 13′, and at the bottom of the house it was 23° 19′. Find the height of the steeple.

215. In walking towards a certain object, I found the angle of elevation of its top to be 2° 19′ 13″ at one milestone, and after proceeding to the next milestone, I found the angle of elevation to be 3° 28′ 49″. How much further should I have to walk before I reached it?

216. Wishing to ascertain the height of a house standing on the summit of a hill, I descended the hill for 40 feet, and then found the height subtended an angle of 34° 18′ 19″. On descending still further a distance of 60 feet, I found the same angle to be 19° 14′ 52″. Find the height of the house.

217. Wanting to know the height of a castle on a rock, I measured a base line of 100 yards, and at one extremity found the angle of elevation of the castle's top to be 45° 15′, and the angle subtended by the castle's height to be 34° 30′; also the angle subtended by the top of the castle and the other extremity of the base line to be 73° 14′. At the other extremity the angle between the first extremity and the top of the castle was 73° 18′. Find the height of the castle.

218. Having measured a base line of 400 yards, whose upper end was 24 feet higher than the lower one, in the same vertical plane with the top of a hill, I found the angles of elevation of the top of the hill from the lower and upper ends of the base line to be 5° 14′ and 3° 17′ respectively. Find the height of the hill.

219. In order to measure the distance between two inaccessible objects C and D, I measured a base line AB of 500 yards, and at its extremities determined the following angles: CAB=94° 13′, DAB=62° 20′, DBA=84° 58′, and CBA=41° 16′. Find the distance between C and D.

220. Wanting to know my distance from an object P on the other side of a river, and having no instrument for observing angles, I measured a base line AB of 500 yards, and from A and B measured directly in a line away from P distances of 175 yards to C and D; I then found my distances from B and A to be respectively, CB = 500 and AB = 650 yards. Find PA and PB.

221. A lighthouse was observed from a ship to bear N. 34° E., and after the ship had sailed due south for 3 miles, the same lighthouse bore N. 23° E. Find the distance of the lighthouse from each position of the ship.

222. Two objects, A and B, were observed from a ship to be at the same instant in a line with a bearing N. 15° E. The ship then sailed N.W. for 5 miles, when it was observed that A bore E., and B bore N.E. Find the distance between A and B.

223. A privateer is lying 10 miles S.W. of a harbour, and observes a merchantman leave it in the direction of S. 80° E., at the rate of 9 miles an hour. In what direction, and at what rate, must the privateer sail in order to come up with the merchantman in $1\frac{1}{2}$ hours?

224. From the top of the peak of Teneriffe, the dip of the horizon is found to be $1° 58' 10''$. If the radius of the earth be 4000 miles, what is the height of the mountain?

225. What is the dip of the horizon from the top of a mountain $1\frac{3}{4}$ miles high, the radius of the earth being 4000 miles?

226. From the top of a mountain $1\frac{1}{2}$ miles high, the dip of the horizon was found to be $1° 34' 30''$. What is the diameter of the earth?

227. In a town are three remarkable objects, A, B and C, known to be distant from each other as follows: $AB = 426{\cdot}75$, $AC = 610$, and $BC = 538{\cdot}5$. From my position S I observe that B lies beyond the line AC, and within the angle ASC: and I find the angle $ASC = 23° 9'$, and $ASB = 14° 16'$. Find the distance of S from A, B and C respectively.

228. Having removed to the other side of the town, so that B lies on the side of AC next to S, and still within the angle ASC; I observe the angle ASC to be $15° 14'$ and $14° 15'$. Find SA, SB, and SC.

229. Having again moved so as to have A and C in a line with S (A being the nearer), I find the angle ASB to be $18° 17'$. Find SA, SB, and SC.

230. Three points of land, A, B and C, are at known distances from each other, namely $AB = 63$, $AC = 44$, and $BC = 76$. At a boat in the piece of water between them the angles subtended by AB and BC are observed to be $89° 15'$ and $130° 45'$ respectively. Find the distance of the boat from A, B and C.

231. Being on a river, and observing a column on the banks, I

find the angle of elevation of its top to be 30°, and the angle subtended by its top and a small island down the river to be 47° 25′. After sailing past the column to this island, a distance of 450 yards, I find the angle subtended by the top and my former position to be 18° 30′. Find the height of the column.

232. On the opposite bank of a river to that on which I stood, is a tower 216 feet high. With a sextant I ascertained the angle subtended at my eye by the height of the tower to be 47° 56′. Find my distance from the foot of the tower, supposing my eye to be 5 feet above the level of the tower's foot.

EXPANSIONS, SERIES, ETC.

233. Expand $(\cos\theta)^6$ in terms of the cosines of θ and its multiples.

234. Expand $(\cos\theta)^{2n}$ in terms of the cosines of multiples of θ. Thence find $(\cos\theta)^2$, $(\cos\theta)^4$, $(\cos\theta)^6$, $(\cos\theta)^8$, $(\cos\theta)^{10}$, $(\cos\theta)^{12}$.

235. Expand $(\cos\theta)^{2n+1}$ in terms of the cosines of multiples of θ. Thence find $(\cos\theta)^3$, $(\cos\theta)^5$, $(\cos\theta)^7$, $(\cos\theta)^9$, $(\cos\theta)^{11}$.

236. Expand $(\sin\theta)^{4m}$ in terms of the cosines of multiples of θ. Thence find $(\sin\theta)^4$, $(\sin\theta)^8$, $(\sin\theta)^{12}$.

237. Expand $(\sin\theta)^{4m+1}$ in terms of the sines of multiples of θ. Thence find $(\sin\theta)^5$, $(\sin\theta)^9$.

238. Expand $(\sin\theta)^{4m+2}$ in terms of the cosines of multiples of θ. Thence find $(\sin\theta)^2$, $(\sin\theta)^6$, $(\sin\theta)^{10}$.

239. Expand $(\sin\theta)^{4m+3}$ in terms of the sines of multiples of θ. Thence find $(\sin\theta)^3$, $(\sin\theta)^7$, $(\sin\theta)^{11}$.

240. Find $\sin 4\theta$, and $\sin 9\theta$ in terms of the powers of $\sin\theta$ and $\cos\theta$.

241. Find $\cos 5\theta$, $\cos 6\theta$ and $\cos 7\theta$ in terms of the powers of $\sin\theta$ and $\cos\theta$.

242. Find $\tan 3\theta$, and $\tan 8\theta$ in terms of the powers of $\tan\theta$.

From the exponential expressions for $\sin\theta$ and $\cos\theta$, obtain the following formulæ:—

243. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

244 $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

245. $\sin\theta = (1-\cos\theta).\cot\dfrac{\theta}{2}$.

246. $\sin 2\theta = 2\sin\theta.\cos\theta$.

247. $\cos\theta = \sqrt{\dfrac{1+\cos 2\theta}{2}}$.

248. $\cot\theta = \dfrac{\sin 2\theta}{1-\cos 2\theta}$.

---

249. Sum the series $1+\cos x+\cos 2x+\cos 3x+\ldots$ to n terms.

250. Sum the series $1+x\cos\theta+x^2\cos 2\theta+x^3\cos 3\theta+\ldots$ to n terms.

251. Sum the series $\sin a + \sin 2a + \sin 3a + \ldots\ldots$ to n terms.

252. Sum the series $\cos\theta + \cos 3\theta + \cos 5\theta + \ldots\ldots$ to infinity.

253. Sum the series $\tan\theta + \frac{1}{2}.\tan\frac{\theta}{2} + \frac{1}{2^2}.\tan\frac{\theta}{2^2} + \frac{1}{2^3}.\tan\frac{\theta}{2^3}$ $+ \ldots\ldots$ to n terms.

254. Sum the series $\tan\theta + 2\tan 2\theta + 2^2\tan 4\theta + \ldots\ldots$ to n terms.

255. Sum the series $\operatorname{cosec} a + \operatorname{cosec} 2a + \operatorname{cosec} 4a + \operatorname{cosec} 8a + \ldots\ldots$ to n terms.

256. Sum the series $1 + \cos\theta.\cos\phi + \cos^2\theta.\cos 2\phi + \cos^3\theta.\cos 3\phi$ $+ \ldots\ldots$ to infinity.

257. Sum the series $\cos\theta + \cos(\theta+\phi) + \cos(\theta+2\phi) + \cos(\theta+3\phi)$ $+ \ldots\ldots$ to $n+1$ terms.

258. Sum the series $\sin\theta - \frac{1}{2}\sin 2\theta + \frac{1}{3}\sin 3\theta - \frac{1}{4}\sin 4\theta + \ldots\ldots$ to infinity.

259. Sum the series $\cos^2\theta + \cos^2(\theta+\alpha) + \cos^2(\theta+2\alpha)$ $+ \cos^2(\theta+3\alpha) + \ldots\ldots$ to n terms.

260. Sum the series $\sin 45^\circ.\sin\theta + \sin^2 45^\circ.\sin 2\theta$ $+ \sin^3 45^\circ.\sin 3\theta + \ldots\ldots$ to n terms.

261. Sum the series $(\cos\theta + \sqrt{-1}\sin\theta) + (\cos\theta + \sqrt{-1}\sin\theta)^2$ $+ (\cos\theta + \sqrt{-1}\sin\theta)^3 + \ldots\ldots$ (1) to n terms, and (2) to infinity.

262. Sum the series $(3^{\frac{1}{2}} - 3^{-\frac{1}{2}}) - \frac{1}{3}(3^{\frac{3}{2}} - 3^{-\frac{3}{2}}) + \frac{1}{5}(3^{\frac{5}{2}} - 3^{-\frac{5}{2}})$ $- \ldots\ldots$ to infinity.

263. Prove that $\tan n\theta = \dfrac{\sin\theta + \sin 3\theta + \sin 5\theta + \ldots\ldots \text{ to } n \text{ terms}}{\cos\theta + \cos 3\theta + \cos 5\theta + \ldots\ldots \text{ to } n \text{ terms}}$, whether n be even or odd.

264. If $\sin x = n\sin(a+x)$; express x in a series of sines of a and its multiples.

265. If $\tan\theta = \dfrac{m\cos\phi}{1+m\sin\theta}$; express θ in a series of sines and cosines of multiples of ϕ.

266. If a and b be two sides of a triangle, opposite to the angles A and B, then

$$\log_e\frac{a}{b} = (\cos 2\mathrm{B} - \cos 2\mathrm{A}) + \frac{1}{2}(\cos 4\mathrm{B} - \cos 4\mathrm{A})$$
$$+ \frac{1}{3}(\cos 6\mathrm{B} - \cos 6\mathrm{A}) + \cdots\cdots$$

267. In any triangle ABC, show that

$$\log_e c = \log_e a - \frac{b}{a}\cos\mathrm{C} - \frac{b^2}{a^2}\cos 2\mathrm{C} - \frac{b^3}{a^3}\cos 3\mathrm{C} - \cdots\cdots$$

268. If $\tan\theta=\frac{b}{a}$, show that

$$\sqrt[n]{a\pm b\sqrt{-1}}=(a^2+b^2)^{\frac{1}{2n}}\left\{\cos\frac{\theta}{n}\pm\sqrt{-1}\sin\frac{\theta}{n}\right\}.$$

269. Show that $\log_e\sec\theta=\frac{\tan^2\theta}{2}-\frac{\tan^4\theta}{4}+\frac{\tan^6\theta}{6}-\ldots.$

thence sum the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots.$ to infinity.

270. If m be any odd number, show that

$$\tan\frac{\pi}{4m}\cdot\tan\frac{5\pi}{4m}\cdot\tan\frac{9\pi}{4m}\cdots\tan\frac{(4m-3)\pi}{4m}=\pm 1,$$

according as m is of the form $4n+1$, or $4n+3$.

271. If a quadrantal arc be divided into two parts in the ratio of $2n:1$, of which θ is the less, prove that

$$\sin\theta.\sin 3\theta.\sin 5\theta\ldots.\sin(2n+1)\theta=\frac{1}{2^n}$$

for all values of n, whether even or odd.

SOLUTION OF CERTAIN EQUATIONS BY TRIGONOMETRY.

Solve the following equations.

272. $x^3-6x=4$.
273. $x^3-147x-343=0$.
274. $x^3-3x+1=0$.
275. $x^3-3x-2=0$.
276. $x^3+1=0$.
277. $x^4+1=0$.
278. $x^5-1=0$.
279. $x^6+1=0$.
280. $x^8-1=0$.
281. $x^6-x^3+1=0$.

SUBSIDIARY ANGLES.

Adapt to logarithmic computation, by means of subsidiary angles, the following formulæ, where the value of x is required.

282. $x=a^2+b^2$.
283. $x=\sqrt{a-b}+\sqrt{a+b}$.

284. $x=\frac{\sqrt{a-b}}{a+b}+\frac{\sqrt{a+b}}{a-b}$.

285. $\cos x=\cos c.\sin A.\sin B-\cos A.\cos B$.

286. $x=\sqrt{a^2+b^2-2ab\cos C}$.

287. $a\sin x+b\cos x=c$.

ANALYTICAL GEOMETRY OF TWO DIMENSIONS.

APPLICATION OF ALGEBRA TO GEOMETRY.

1. To describe a square in a given triangle.

2. Given the hypothenuse of a right-angled triangle, and the side of an inscribed square: required the two sides of the triangle, (1) when the given side coincides with the hypothenuse, and (2) when it does not so coincide.

3. In a given square to inscribe another square having its side equal to a given straight line. To what limitation is this line subject?

4. To find a triangle such that its sides and a perpendicular on one of them from the opposite angle are in continued geometrical progression.

5. Through a point M equidistant from two straight lines AA′, BB′ at right angles to each other, to draw a straight line PMQ, so that the sum of the squares upon PM and MQ shall be equal to the square upon a given line b.

6. To inscribe a semicircle in a quadrant.

7. To inscribe a circle in a given sector of a circle.

8. If a, b, c be the chords of three adjacent arcs of a circle whose sum equals the semicircumference, of which x is the radius, then will $4x^3-(a^2+b^2+c^2)x-abc=0$.

9. Given the three perpendiculars from the angles of a triangle upon the opposite sides; to find the area and the sides of the triangle.

10. Given the chords of two arcs of a given circle; to find the chord of their sum, and the chord of their difference.

11. Having given the lengths of two chords which intersect at right angles, and the distance of their point of intersection from the centre; to find the diameter of the circle.

12. If r be the radius of a circle which touches the side a of a triangle, and the other two sides produced, then $r\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)=a$, B and C being angles of the triangle at the extremities of a.

13. The radii of two circles which intersect one another are r, r'; and the distance of their centres is c; find the length of their common chord.

14. What plane rectilineal figure is that in which a similar rec-

tilineal figure can be inscribed whose area shall equal half that of the original?

15. If from one of the angles of a rectangle a perpendicular be drawn to its diagonal, and from the point of their intersection lines be drawn perpendicular to the sides which contain the opposite angle; show that, if p, p' be the lengths of the perpendiculars last drawn, and d the diagonal of the rectangle, $p^{\frac{2}{3}}+p'^{\frac{2}{3}}=d^{\frac{2}{3}}$.

> In solving this question (1) show that the adjacent sides a and b of the rectangle are to one another as $\sqrt[3]{p}:\sqrt[3]{p'}$; then (2) eliminate a and b from the equation which this proportion gives, by the aid of two equations, one of which involves only a, b and d; the other a, b, p and p'.

16. In a given square a circle is inscribed; in one of the interstices between this circle and square another circle is inscribed; and in the interstice between the second circle and square a third circle is inscribed, and so on to infinity; prove that the sum of the circular areas $=\frac{\pi a^2}{4}\left(\frac{3+2\sqrt{2}}{4\sqrt{2}}\right)$, where $a=$ side of square.

> In this question first show that the radii of the successive circles are in geometrical progression, having $\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$ for the common ratio. The infinite geometric series expressing the circular areas may then be formed and summed.

17. Into a hollow cone whose vertical angle is 2α are put a number of spheres one above another, each sphere touching the one above, the one below, and the sides of the cone; show that the radii of the spheres form a geometrical progression, of which the common ratio is $\tan^2\left(45^\circ\pm\frac{\alpha}{2}\right)$.

18. If r be the radius of the circle inscribed between the base of a right-angled triangle and the other two sides produced, and r' be the radius of the inscribed circle between the altitude of the same triangle and the other two sides produced; the area of the triangle $=rr'$.

> This theorem may readily be demonstrated by the aid of the property enunciated in Ex. 12.

19. If r and r' be the radii of two circles in one plane, and a the distance between their centres, and if the line joining those centres be produced indefinitely, the two points in this line, from each of which the two circles will appear equally large, are at a distance from each other equal to $\frac{2arr'}{r'^2-r^2}$; and if a circle be described upon the line joining these two points as a diameter, the two original

circles will appear equally large from every point in the circumference of this third circle.

The latter part of this proposition may be proved by the aid of Prop. LXXXI. Geometry in Cape's Course of Mathematics.

20. If $y=mx$ be the equation to a chord of the circle whose radius is r; show that the equation to the circle of which that chord is a diameter is $(1+m^2)(x^2+y^2)-2r(x+my)=0$.

21. The area included between the straight lines, of which the equations are $y=x\tan\alpha$, $y=x\tan\alpha_1$ and $y=x\tan\alpha_2+a$, is equal to $\frac{a^2}{2}\cdot\frac{\sin(\alpha-\alpha_1)\cos^2\alpha_2}{\sin(\alpha-\alpha_2)\sin(\alpha_1-\alpha_2)}$.

22. The diameter of a semicircle is divided into two parts, on each of which as diameter a semicircle is described, and on the same side as the given one; if a circle be described touching each of these three semicircles, the distance of its centre from their common diameter is equal to twice its radius.

STRAIGHT LINE.

23. Find the equation to a straight line which lies evenly between two given parallel straight lines.

24. Determine the point of intersection of the two lines whose equations are $y=mx+c$ and $y=-\frac{1}{m}x+c'$.

25. Show that the equation $(2y+x)(3y-x)=0$ represents two straight lines inclined to one another at 135° or 45°.

26. The equation $2y^2-3xy-2x^2-y+2x=0$ represents two straight lines at right angles to one another.

27. Find the polar equation to a straight line; and trace the line whose equation is $r=2a\,.\cos\left(\theta+\frac{\pi}{6}\right)$.

28. Find the polar equation to a line which shall pass through two points whose polar coordinates are given.

29. Find the equation to a straight line passing through a given point in the axis of x, and making an angle of 45° with the straight line whose equation is $\frac{x}{a}-\frac{y}{b}=1$.

30. Find the equation to a straight line perpendicular to a given straight line, and cutting the axis of y in a given point.

31. Find the equation to a straight line which passes through the point $x=1$, $y=3$, and makes an angle of 30° with the straight line whose equation is $2y-x+1=0$.

32. Find the cosine of the angle contained between two straight lines whose equations are $y-3x+c=0$, and $y-5x+d=0$.

33. Find the equation to a straight line which shall pass through the point of intersection of the two lines whose equations are $5y-2x=0$, and $3y+4x-12=0$; and shall bisect the angle between them.

34. What is the inclination of the axes when the lines represented by the equation $y^2-x^2=0$ are perpendicular to one another?

35. Find the equation to a line touching a given circle, and parallel to a given straight line.

36. In the figure to Euclid, Book I. Prop. XLVII., prove that FC, KB and AL intersect each other in the same point.

> N.B. In the Geometry of Cape's Course of Mathematics the corresponding proposition and lines are LXII., HC, IB and AE respectively.

37. In the equation $ay^2+bxy+dy+ex=0$, find the relation among the coefficients that it may represent two straight lines.

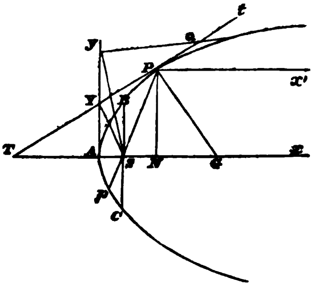
CONIC SECTIONS.

PARABOLA.

General Properties.

If $AS=a$, $AN=x$, $NP=y$, then

1. $y^2=4ax$.
2. BC the latus rectum $=4a$.
3. $SP=a+x$.
4. $yy'=2a(x+x')$ is the equation to the tangent Tt.
5. TN the subtangent $=2x$.
6. NG the subnormal $=2a$.
7. $SY^2=ST\times SA=SP\times SA$.
8. $<SPT=<tPx'$.
9. Area $APN=\frac{2}{3}AN\times PN$.



If $SP=r$, $<ASP=\theta$, then

10. $r=\dfrac{2a}{1+\cos\theta}$.

38. If PSp be any parameter of a parabola whose focus is S and latus rectum L, prove that

$$4SP\,.\,Sp=L(SP+Sp).$$

39. Two tangents to a parabola drawn from the same point of the directrix are at right angles to each other.

40. Show that there is only one point in the parabola at which the focal distance is perpendicular to the tangent.

In this case the property (7) may be used.

41. Draw a normal at the extremity of the latus rectum of a parabola whose equation is $y^2=4a(x-a)$, and find its distance from the origin of coordinates.

42. The tangent at any point of a parabola will meet the directrix and latus rectum produced, in two points equally distant from the focus.

43. If two equal parabolas have a common axis, a straight line touching the interior parabola, and bounded by the exterior, will be bisected by the point of contact.

44. Given the radius vector at any point of a parabola and the angle it makes with the curve; find the latus rectum and the position of the vertex.

45. Show that the parameter belonging to any diameter of a parabola varies inversely as the square of the sine of the angle at which the corresponding ordinates are inclined to it.

The polar equation should be assumed in this case.

46. Describe a parabola which shall touch a given circle at a given point, and have its axis coincident with a given diameter of the circle.

In this case the latus rectum and the position of the vertex of the parabola must be found. The tangent to the circle at the given point is also a tangent to the parabola at that point. The extremity of the given diameter of circle may conveniently be taken for the origin of coordinates.

47. The abscissa and double ordinate of a parabola are h and k; the diameters of its circumscribed and inscribed circles are D and d; prove that $D+d=h+k$.

In solving this question we find that $D=4a+h$ and $d=-4a+k$.

48. If PQ be the chord of a parabola, which is a normal at P, and the tangents at P and Q intersect in a point T; prove that the line PT is bisected by the directrix.

(1.) Having given the coordinates of P, find those of Q by combining the equations to the normal at P and to the parabola.
(2.) Find the coordinates of T, by combining the equations to the tangents at P and Q.

49. If BC, CD be two consecutive arcs of a parabola, the sagittæ of which drawn parallel to the axis are equal; prove that the chord of BCD is parallel to the tangent at C.

Def. The sagitta of an arc is a straight line drawn from the middle point of the chord to meet the arc.
(1.) It must be shown that the chord of BCD is bisected by the line drawn through C parallel to the axis.
(2.) If α, β be the angles at which the chords BC, CD respectively are inclined to the axis, then $BC \sin\alpha = CD \sin\beta$.

50. If AB, BC be two consecutive arcs of a parabola, and LW, GH, MV be the sagittæ, drawn parallel to the axis of the arcs AB, AC, BC respectively; prove that $\sqrt{LW}+\sqrt{MV}=\sqrt{GH}$.

The method used in the solution of the preceding question may here be repeated.

51. Find the position and magnitude of the parabola whose equation is $\sqrt{\dfrac{x}{a}}+\sqrt{\dfrac{y}{b}}=1$.

52. Find the area included between the parabola $y^2=4ax$, and the straight line $x=y+a$.

53. A triangle is formed by the meeting of three tangents to a parabola. Show that the products of the alternate segments of the tangents made by their mutual intersections are equal.

This question may be solved by the following property: if P, Q be points in the parabola at which two of the tangents are drawn, S the focus, and B the intersection of these tangents, then $PB^2 : QB^2 = SP : SQ$.

54. A circle described through the intersections of three tangents of a parabola will pass through the focus.

If lines be drawn from the focus to the extremities of that tangent which lies between the other two, the angle at the focus is the supplement of the angle between those two tangents. As in the last question $PB^2 : QB^2 = SP : SQ$; also $SP . SQ = SB^2$: both these properties are required for this problem.

55. If one side of a triangle and two others produced be tangents to a parabola, and the points of contact be joined, a triangle will be formed whose area is double of that of the exterior triangle.

56. From the vertex of a parabola a straight line is drawn, inclined at 45° to the tangent at any point; find the equation to the curve which is the locus of their intersections.

The equation to the curve is $a(y+x)^2 = (y^2+x^2)(y-x)$.

57. In the focal distance SP take Sp equal to the ordinate PN. Find the equation to the curve traced out by the point p.

If we assume $Sp = \rho$ and $\angle NSp = \theta$, the locus of p is expressed by the equation $\rho = 2a . \cot\frac{\theta}{2}$.

58. Two straight lines which are always tangents to a given parabola, are so inclined to the axis of x that the sum of the cotangents of the angles which they make with that axis is constant; prove that the locus of their intersections is a straight line parallel to the axis.

59. If several parabolas have the same vertex and axis, the locus of the extremities of normals to them from a given point in the axis is an ellipse.

60. From any point Q in the line BQ which is perpendicular to the axis CAB of a parabola whose vertex is A, QP is drawn parallel to the axis to meet the curve in P; show that if CA be taken equal to AB the locus of the intersections of AQ and CP is a parabola.

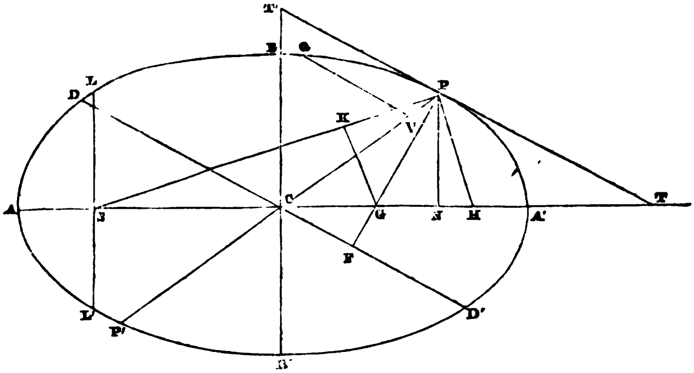
The required locus coincides with the original parabola.

61. If PT, QT be two tangents at the points P, Q of a parabola whose focus is S, then $SP . SQ = ST^2$; and if SP, SQ include a given angle α, the locus of T will be a hyperbola whose eccentricity is equal to $\sec\frac{\alpha}{2}$.

62. The locus of the intersection of two normals to a parabola at right angles to one another, is a parabola whose latus rectum is one-fourth of the latus rectum of the original.

ELLIPSE.

General Properties.



If $CA=a$, $CB=b$, $\frac{CS}{CA}=e$; $CN=x$, $NP=y$; then

1. $y^2=\frac{b^2}{a^2}(a^2-x^2)$.
2. LL' the lat. rect. $=\frac{2b^2}{a}$ or $2a(1-e^2)$.
3. $SP=a+ex$, $HP=a-ex$, S, H being the foci.
4. $a^2yy'+b^2xx'=a^2b^2$ is the equation to the tangent at P.
5. $CT=\frac{a^2}{x}$, $CT'=\frac{b^2}{y}$.
6. $CG=e^2x$, $GN=\frac{b^2}{a^2}x$, PG being the normal at P.
7. $QV^2=\frac{CD^2}{CP^2}\cdot PV.VP'$, where QV and CD are parallel to PT.
8. $CP^2+CD^2=a^2+b^2$ and $CD^2=SP.PH$.
9. $CD.PF=ab$.
10. Area of ellipse $=\pi ab$.

If $SP=r$, $\angle ASP=\theta$, then

11. $r=\frac{a(1-e^2)}{1+e\cos\theta}$.

If $CP=r$, $\angle ACP=\theta$, then

12. $r=\dfrac{b}{\sqrt{1-e^2\cos^2\theta}}$.

If ρ be the radius of curvature at P, then

13. $\rho=\dfrac{\mathrm{CD}^3}{ab}=\dfrac{a^2}{b^4}\times \mathrm{PG}^3$.

63. If A, S, C be the vertex, focus and centre of an ellipse; it is required to show that if SC become infinite, AS remaining finite, the ellipse will be changed into a parabola.

64. In an ellipse find the position of that focal distance SP which is a mean proportional to the semi-axes; when $a=50$, $b=30$.

65. If in any ellipse there be taken three abscissas in arithmetical progression; the radius-vectors drawn from the focus to the extremities of the ordinates at those points will also be in arithmetical progression.

For this question use formula (3).

66. In an ellipse whose semi-axes are 5 and 4, find the position of CP when an arithmetic mean between CA and CB.

67. At what point in an ellipse is the angle, formed by the two focal distances, greatest?

68. In any ellipse show that $\tan\dfrac{\mathrm{PSH}}{2}\,.\,\tan\dfrac{\mathrm{PHS}}{2}=\dfrac{1-e}{1+e}$.

69. If PQ be two points in an ellipse, such that the lines CP, CQ are at right angles to each other, then will $\dfrac{1}{\mathrm{CP}^2}+\dfrac{1}{\mathrm{CQ}^2}=\dfrac{1}{a^2}+\dfrac{1}{b^2}$.

Formula (12) will here be of service.

70. If CP be any semidiameter of an ellipse, and AQO be drawn from the extremity of the major axis parallel to CP, and meeting the curve in Q and the minor axis produced in O; show that $2\mathrm{CP}^2=\mathrm{AO}\,.\,\mathrm{AQ}$.

Find the ordinates to Q and O, by means of the equations to the straight line AQO and to the ellipse; then by similar triangles express these ordinates in terms of AO, AQ and CP.

71. The length of the perpendicular upon the tangent from the centre of an ellipse is equal to $a\sqrt{1-e^2\cos^2\phi}$, where ϕ is the inclination of the tangent to the axis major.

72. If λ be the acute angle between the tangent and focal distance at any point of an ellipse, the distance of that point from the centre is equal to $\sqrt{a^2-b^2\cot^2\lambda}$.

First show that $\mathrm{CP}^2=a^2+e^2x^2-a^2e^2$; then find a relation between x and λ from the equation $\dfrac{\mathrm{PK}}{\mathrm{PG}}=\sin\lambda$ (see the figure); this gives $b^2\cot^2\lambda=a^2e^2-e^2x^2$.

73. If from G the foot of the normal at P a perpendicular GK be drawn to either focal distance, then PK will equal half the latus rectum.

74. If PG, PG′ be parts of the normal cut off by the axes major and minor respectively, prove that $\frac{\text{PG}}{\text{PG}'}=\frac{b^2}{a^2}$.

75. Draw a tangent at the extremity of the latus rectum of the ellipse whose equation is $\frac{x^2}{9a^2}+\frac{y^2}{4a^2}=1$.

76. A tangent at L (see figure) meets any ordinate NP produced in R; show that NR = SP.

77. A tangent at the extremity of the latus rectum intersects the minor axis produced in the circumference of a circle on the major axis.

78. In an ellipse the tangents at the extremities of any focal chord intersect in the directrix and in a perpendicular to the chord from the focus.

79. If a line be drawn through the focus of an ellipse, making an angle θ with the major axis, and tangents be drawn at the extremities of this line, these tangents will be inclined to one another at an angle ϕ, such that $\tan\phi=\frac{2e}{1-e^2}\sin\theta$.

80. The centre of an ellipse coincides with the vertex of a common parabola, and the axis major of the ellipse is perpendicular to the axis of the parabola. Required the proportion of the axes of the ellipse, so that it may cut the parabola at right angles.

> A tangent to the parabola at the point of its intersection with the ellipse is a normal to the ellipse at the same point. Hence we may show that $\frac{a}{b}=\sqrt{2}$.

81. If with the coordinates of any point in an elliptic quadrant as semi-axes, a concentric ellipse be described, the chord of the quadrant of the former will be a tangent to the latter.

> The equation to the chord is $\frac{x}{a}+\frac{y}{b}=1$; and the equation to the tangent of the latter ellipse may be rendered identical with it by property (5).

82. If α, β; α', β' be the coordinates of two points in a diameter of an ellipse, and be subject to the condition $a^2b^2=a^2\beta\beta'+b^2\alpha\alpha'$, show that the equations to the tangents at the extremity of this diameter are $a^2\sqrt{\beta\beta'}\,.\,y+b^2\sqrt{\alpha\alpha'}\,.\,x=\pm a^2b^2$.

83. In a given equilateral parallelogram inscribe an ellipse of given eccentricity.

84. If ABC be a given triangle, show how to find the ellipse which shall have A and B for its foci, and shall touch a circle of given radius whose centre is C.

85. Through any point in the line joining the centre and intersection of the tangents to any two points of an ellipse, two straight lines are drawn respectively parallel to its diameters passing through the points of contact; prove that the triangles formed by these lines and the tangents are equal.

86. Find the equation to the normal at any point of an ellipse, expressed in terms of its inclination to the axis major.

87. If θ be the angle which the focal distance to any point of an ellipse makes with the tangent, and ϕ the angle between the lines drawn from that point to the extremities of the axis major, then $2 \tan \theta = e \tan \phi$.

88. In any ellipse prove that GK (see figure) is equal to $e \, . \, \text{PN}$.

89. Find expressions for the chords of curvature through the focus and centre of an ellipse.

90. If e, e' be the radii of curvature at the extremities of two conjugate diameters of an ellipse, show that $\varrho^{\frac{2}{3}} + \varrho'^{\frac{2}{3}}$ is constant. Also if c, c' be the curvatures at two points at which the tangents are at right angles, $c^{\frac{2}{3}} + c'^{\frac{2}{3}}$ is constant.

For the former part of this question use formula (13), and for the latter it may be sufficient to remark, that the curvature at any point of a curve varies inversely as the radius of curvature at that point.

91. Find the length of the normal at any point of an ellipse expressed in terms of its inclination to the axis major.

92. If a right line be drawn from the extremity of any diameter of an ellipse to the focus, the part intercepted by the conjugate diameter is equal to the semi-axis major.

93. Two conjugate diameters of an ellipse include an angle γ; show that these diameters are equal to one another when $\sin \gamma$ is the least possible. In this case find the value of γ; a being 8 and b, 5.

The value of γ is given by the equation $\tan \frac{\gamma}{2} = \frac{5}{8}$.

94. If from the extremities of any diameter of an ellipse chords be drawn to any point in the curve, and one of them be parallel to a diameter, the other will be parallel to the conjugate diameter.

95. The diameters which bisect the lines joining the extremities of the axes of an ellipse are equal and conjugate.

In this question formula (12) will be found convenient.

96. If $\text{CP} = a'$, and CD the semi-conjugate to $\text{CP} = b'$ have such

a position that $<A'CP=\alpha$, and $DCB=\beta$; show that

$$\frac{a'^2-b'^2}{a^2-b^2}=\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}.$$

If x', y' be the coordinates of P, we may show that

$$\frac{a'^2-b'^2}{a^2-b^2}=2\frac{x'^2}{a^2}-1=\frac{x'^2}{a^2}-\frac{y'^2}{b^2}=\frac{CN}{CT}-\frac{PN}{CR}.$$

97. A triangle is described about an ellipse; prove that the products of the alternate segments of the sides made by the points of contact are equal.

The segments of tangents drawn from the same point without the ellipse are proportional to the diameters respectively parallel to them.

98. If a polygon circumscribe an ellipse, the continued products of the alternate segments are equal to one another.

The remark made in the last question may here be repeated.

99. If SP, HP be any two focal distances in an ellipse whose vertex is A; and if AQL be drawn cutting SP in Q and bisecting HP in L; show that the locus of Q is an ellipse whose axes are $2a(1-e)$ and $2b(1-e)$.

If S be taken for the origin of coordinates, x, y the coordinates of Q, $x'y'$ those of P, then we may show that $x'=\frac{2x}{1-e}$ and $y'=\frac{2y}{1-e}$; which results lead at once to the solution.

100. If C be the centre of an ellipse, and in the normal to any point P, PQ be taken equal to the semi-conjugate at P; show that the locus of Q is a circle of which C is the centre.

It may be shown that $CQ=a-b$, a constant quantity, and therefore the radius of a circle about C.

101. If SQ be drawn always bisecting the angle PSC of an ellipse (see figure) and equal to the mean proportional between SC and SP; find the eccentricity of the locus of Q.

Either polar or rectangular coordinates may be used; and the result gives the eccentricity required $=\sqrt{\frac{2e}{1+e}}$.

102. CP and CD are semi-conjugate diameters of an ellipse, and PF is a perpendicular let fall upon CD or CD produced; determine the locus of the point F.

The equation to the locus of F is found by eliminating the coordinates of P from the equations to CD, PF and the ellipse. The result is

$$\left(\frac{x^2+y^2}{a^2-b^2}\right)^2\left(\frac{a^2}{x^2}+\frac{b^2}{y^2}\right)=1.$$

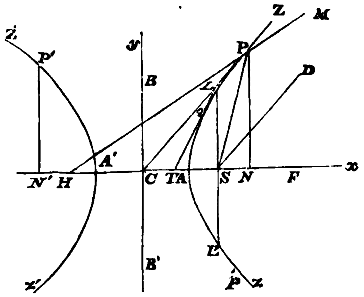
103. Find the equation to the curve from any point of which if two tangents be drawn to a given ellipse the angle contained between them shall be constant.

104. Prove that the locus of the points of bisection of any number of chords of an ellipse which pass through the same point is an ellipse; and find the magnitude and position of the axes when the coordinates to the point are given.

The eccentricity of this locus is the same as that of the original ellipse.

105. If tangents drawn to any two points of an ellipse meet each other; show that their lengths are inversely as the sines of the angles which they make with the lines drawn to either focus.

HYPERBOLA.



The general formulæ expressing the properties of the hyperbola are similar to those of the ellipse. So that if any result be obtained for the ellipse in terms of its axes, the corresponding result for the hyperbola may be obtained by writing $b\sqrt{-1}$ for b. We refer therefore to the formulæ for the ellipse, which, though not identical with those for the hyperbola, will yet serve to suggest them. Many of the foregoing problems might easily, for the same reason, be modified to suit the hyperbola; and hence it is not necessary to repeat them.

106. The tangent to any point of a hyperbola is produced to meet the asymptotes; show that the triangle cut off is of constant magnitude.

The equation to the hyperbola referred to its asymptotes is $xy=\frac{1}{4}(a^2+b^2)$. Hence the equation to the tangent at the point x', y', is $xy'+x'y=\frac{1}{2}(a^2+b^2)$.

107. An ellipse and a hyperbola have the same foci; show that these curves will intersect one another at right angles.

108. Express the length of a perpendicular from the centre of a hyperbola upon the tangent at any point in terms of its inclination to the transverse axis.

109. Find the locus of the extremity of a perpendicular from the centre of a hyperbola upon a tangent at any point.

The equation to this locus is $(x^2+y^2)^2=a^2x^2-b^2y^2$.

110. Find the locus of the intersection of two tangents to a hyperbola which meet one another at right angles.

This locus is a circle whose radius is $\sqrt{a^2-b^2}$.

111. A pair of conjugate hyperbolas being given, to find their centre.

112. Investigate the polar equation to the hyperbola, the focus being the pole, having given SP − HP = 2AC; and draw the asymptote by means of this equation.

113. If 3CA = 2CS in a given hyperbola, find the inclination of the asymptote to the transverse axis.

114. Of two conjugate diameters of a hyperbola, one only meets the curve; and if one be drawn through a given point of the curve, find where the other meets the conjugate hyperbola.

115. If a line intersecting a hyperbola in the point P, and its asymptotes in R, *r*, move parallel to itself, the rectangle RP.P*r* is constant.

116. If A and B be the extremities of the axis major of a hyperbola or an ellipse, T the point where the tangent at P meets AB or AB produced, QTR a line perpendicular to AB and meeting AP and BP in Q and R respectively, then QT = RT.

117. Find the latus rectum and eccentricity of the hyperbola which is conjugate to that whose equation is $y^2=4(x^2+a^2)$.

The latus rectum $=8a$, the eccentricity $=\sqrt{5}$.

118. If normals be drawn to an ellipse from a given point within it, the points where they cut the curve will all lie in a rectangular hyperbola which passes through the given point and has its asymptotes parallel to the axes of the ellipse.

119. Find the radius of curvature to a hyperbola at the extremity of its latus rectum, the axes being 20 and 12.

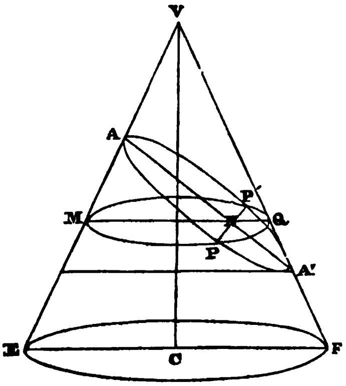
The radius of curvature at any point $=\dfrac{(\text{normal})^3}{(\text{semi-lat. rect.})^2}$.

SECTIONS OF THE CONE AND GENERAL PROBLEMS.

Let $AN=x$, $NP=y$, $AV=d \angle VAN=\theta$, $\angle AVQ=2\alpha$; then will

$$y^2=\frac{2d.\sin\alpha.\sin\theta}{\cos\alpha}.x-\frac{\sin\theta.\sin(2\alpha+\theta)}{\cos^2\alpha}.x^2$$

is the equation to the conic section.



120. A conic section is traced upon a plane; draw a tangent from any point without it by means of a ruler and pencil only.

121. If a right cone whose vertical angle is 90° be cut by a plane which is parallel to one touching the slant side, prove that the latus rectum of the section is equal to twice its distance from the vertex.

122. If a right cone of which the semivertical angle is γ be cut by a plane making an angle δ with its axis, the ellipse thus obtained will be such, that the

$$\text{minor axis} : \text{major axis} = \sqrt{\sin(\delta+\gamma).\sin(\delta-\gamma)} : \cos\gamma.$$

123. The section of a right cone by a plane is an ellipse of which a and b are the axes, h and k the distances from the vertex to the points where the plane cuts the sides of the generating triangle; show that $a^2-b^2=(h-k)^2$.

If 2θ be the vertical angle of cone it may be shown that $a^2=(h-k)^2+4hk.\sin^2\theta$, and $b^2=4hk\sin^2\theta$.

124. When the section of a cone is an ellipse or a hyperbola, show that the semi-minor axis is a mean proportional between the perpendiculars dropped from the extremities of the major axis upon the axis of the cone.

125. Find the latus rectum of the section of a right cone made by a plane parallel to the slant side; and when the plane passing through the directrix of the section, and the vertex of the cone is perpendicular to the cutting plane, determine the vertical angle of the cone.

The vertical angle is equal to $\cos^{-1}\left(\frac{1}{3}\right)$.

126. At what angle must a plane be inclined to the side of a cone in order that the section may be a rectangular hyperbola? Determine also the least vertical angle of the cone for which the problem is possible.

127. Prove that in any conic section the diameter of curvature varies as the cube of the normal.

128. In any conic section the projection of the normal on the focal distance is constant.

129. In any conic section, if S be the focus and G the foot of the normal at P, then SG varies as SP.

130. The sum of the squares of normals at the extremities of conjugate diameters in any conic section is constant.

131. AP is the arc of a conic section of which the vertex is A; PG the normal, and PK a perpendicular to the chord AP meet the axis in G and K respectively. Show that GK is equal to half the latus rectum.

132. If S be the focus and A the vertex of any conic section, and if LT the tangent at the extremity L of the latus rectum meet the axis in T, show that $\frac{\text{AS}}{\text{AT}}$ equals the eccentricity.

133. In any conic section, if two chords move parallel to themselves and intersect each other, the ratio of the rectangles of their segments is invariable.

134. If PSp be any straight line drawn through the focus S of a conic section, meeting the curve in the points P and p, and SL be the semi-latus rectum, then will SP, SL and Sp be in harmonical progression.

135. A straight line drawn from the intersection of two tangents to a curve of the second order is harmonically divided by the curve and the chord joining the points of contact.

136. A conic section is cut in four points by a circle, and two lines each passing through two of the points of intersection are made the axes of coordinates, their point of meeting being the origin; show that the equation to this conic section is of the form

$$x^2+bxy+y^2+dx+ey+f=0.$$

Under what conditions is the converse true?

137. The distance of a point P from the circumference of a circle : its distance from a fixed diameter AB : : n : 1. Prove that the locus of P is a conic section.

> The equation to the locus of P expresses an ellipse or hyperbola, or parabola, according as n is less than, or greater than, or equal to unity respectively.

138. If two lines revolving in the same plane round the points S and H, intersect one another in the point P in such a manner that, (1) SP^2+HP^2 equals a constant quantity; (2) SP is to HP in the given ratio of n : 1; prove that in each case the locus of P is a circle.

139. Find the locus of the intersection of two normals to a curve of the second order, which cut one another at right angles.

If ρ, θ be the polar coordinates, the centre being the pole, the required locus may be expressed by $\frac{\rho}{c}=\frac{1-n^2\tan^2\theta}{1+n^2\tan^2\theta}$, where $n=\frac{a}{b}$ and $c=\frac{a^2-b^2}{\sqrt{a^2+b^2}}$. This result may be obtained through the medium of rectangular coordinates; and the solution commenced by finding the equation to a normal in terms of its inclination to the transverse or major axis.

140. Find the locus of the intersection of two tangents to a curve of the second order, which cut one another at any given angle.

141. Find the equation of the curve traced out by the extremities of the perpendiculars on the tangents of a circle, drawn from a point in its circumference.

142. If the coordinate axes of the curve whose equation is $(x^2-y^2)^2=ax(x^2+3y^2)$ be made to revolve about the origin through an angle of 45°; required the equation to the curve referred to the new axes.

143. Determine the axes to which the rectangular equation $a^2y^2+b^2x^2=a^2b^2$ must be transferred, so that the transformed equation may be $x'^2+y'^2=a'^2$.

Determine the nature and position of the several curves expressed by the following equations:—

144. $7x+2y+14a=0$. 145. $12xy+8x=27y+18$.

146. $x^2+y^2-8y+12x+48=0$.

147. $x^2+y^2-4y+2x+9=0$.

148. $3x^2+2y^2-2x+y-1=0$.

149. $y^2-2xy+x^2-8x+16=0$.

150. $5y^2+2xy+5x^2-12x-12y=0$.

151. $4y^2+4x^2+16y-8x+19=0$.

152. $3x^2+2xy+3y^2-16y+23=0$.

153. $y^2-10xy+x^2+y+x+1=0$.

154. $y^2-2xy+x^2-6y-6x+9=0$.

155. $y^2-2xy+2x^2-2y-4x+9=0$.

156. $y^2+2xy+2x^2-2y+4x+9=0$.

157. $y^2+xy+x^2+y+x-5=0$.

158. $y^2+2xy+3x^2-4x=0$.

159. $y^2-2xy+3x^2-2y-10x+19=0$.

160. $y^2-4xy+5x^2+2y-4x+2=0$.

161. $y^2-3xy+x^2+1=0$, the axes being oblique, and inclined to one another at an angle of 60°.

162. $y^2-2xy-x^2+2=0$. 163. $y^2-x^2-y=0$.

164. $7y^2+16xy+16x^2+32y+64x+28=0$; referred to the axes used in 161.

165. $4y^2-8xy-4x^2-4y+28x-15=0$.

166. $2y^2+x^2+4y-2x-6=0$.

167. $5y^2+6xy+5x^2-22y-26x+29=0$.

168. $y^2-2xy+x^2-2y+2x=0$.

169. $y^2+2xy+x^2+1=0$.

170. $5xy+3x^2-10x+3y-12=0$.

171. $7x(y+4)-6y^2+8y-3=0$.

172. Show that the curve $y=5x-\frac{7}{x}$ has the origin of coordinates for its centre; trace this curve and find the magnitude of its axes.

---

173. Find the equation to the locus of a point the difference of whose distances from two fixed points is invariable; and trace the curve.

174. The base of a triangle is constant, and the sum of the angles at the base is also constant; find the locus of the vertex.

175. The base of a triangle is given, and one angle at the base is double of the angle at the other extremity of the base; find the locus of the vertex.

176. Determine the locus of a point so situated within a plane triangle, that the sum of the squares of the straight lines drawn from it to the angular points is constant; if the curve has a centre, find its position.

177. Having given the base and area of a triangle, find the locus of the centre of the circumscribed circle.

178. Straight lines are drawn from a fixed point to the several points of a line given in position, and on each as a base is described a triangle whose vertical angle is one-half of each of the angles at the base; find the locus of these vertices.

179. The base of a triangle and the sum of the other two sides are given; find the locus of the centre of the inscribed circle.

180. If two lines SP, HP revolve about the points S, H so that $SP \times HP = CS^2$ (C being the middle point of SH); it is required to find the locus of P.

181. If from two fixed points in the circumference of a circle, straight lines be drawn intercepting a given arc and meeting without the circle; to find the locus of their intersection.

182. A straight line revolving in its own plane about a given point intersects a curve line in two points; find the curve when the rectangle of the lines intercepted between the given point and the points of intersection is constant.

183. A straight line of given length $2c$ is made to move so that its ends are always in contact with two other straight lines which

include a given angle 2α; show that the locus of its middle point is an ellipse whose semi-axes are c tan α and c cot α; and the direction of one of its axes bisects the angle included between the two given lines.

184. From two given points A and B two straight lines given in position are drawn; MRQ is a common ordinate to these lines, and MP is taken in MRQ a mean proportional to MQ and MR; required the locus of P.

185. If the base and the difference of the angles at the base of a triangle be given, the locus of the vertex is an equilateral hyperbola.

186. Let AQA′ be an ellipse, AA′ the axis major, QQ′ any double ordinate; join AQ and A′Q′ and produce these lines to intersect one another in P; the locus of P is required.

187. To find the locus of the centres of all the circles drawn tangential to a given line, and whose circumferences pass through a given point.

188. Let AQB be a semicircle of which AB is the diameter, BR an indefinite straight line perpendicular to AB, AQR a straight line meeting the circle in Q and BR in R; take AP = QR; required the locus of P.

The locus of P is called the Cissoid of Diocles.

189. A point Q is taken in the ordinate MP of the parabola, always equidistant from P and from the vertex of the parabola; required the locus of Q.

190. Let AQB be a semicircle in which AB is the diameter and NQ is any ordinate produced to P, so that NP : NQ = AB : AN; to find the locus of P.

The locus of P is called the Witch of Agnesi.

191. Let XX′ be an indefinite straight line, A a given point without it, from which draw the straight line ACB perpendicular to XX′ which it cuts in the point C, and also any number of straight lines AEP, AE′P′, &c.; take EP, always equal to CB; then the locus of P is required.

This locus is called the Conchoid of Nicomedes.

Trace the curves whose equations are,—

192. $r = 2a \,.\, \sin\theta \,.\, \tan\theta.$

193. $r^2 = a^2 \cos 2\theta.$

194. $y^3 = a^2 x - x^3.$

195. $xy = \pm (a + y) \sqrt{b^2 - y}$, when b is greater than a.

196. $y + x = \sqrt{3x}.$

197. $y^2(a^2 - x^2) = x^2(a^2 + x^2).$

198. $y^2 = 3x \pm x\sqrt{9 - x^2}.$

199. $y = \dfrac{x-3}{(x-1)(x-2)}.$

200. $(x^2 + y^2)(x - \sqrt{x^2 - y^2}) = ay^2.$

STATICS.

FORCES ACTING ON ONE POINT AND IN THE SAME PLANE.

The two conditions of equilibrium are,—

1. The sum of the resolved forces in *any* direction $=0$.

2. The sum of the resolved forces in a direction at right angles with the first direction $=0$.

1. Three forces acting on a point, and in the same plane, are in equilibrium when their directions are inclined to each other at angles of 60°, 135°, and 165° respectively. Find the ratio of the forces.

2. Three forces acting on a point, keep it at rest; and they are in the ratios of $\sqrt{3}+1 : \sqrt{6} : 2$. Find the angles at which they are respectively inclined to each other.

3. Three equal forces, each equivalent to 6 lbs., act on a point; the first two are inclined to each other at an angle of 75°, and the third is inclined at an angle of 15° to the first. Find the magnitude and direction of the resultant.

4. Four forces, represented by 1, 2, 3 and 4, act on a point. The directions of the first and third are at right angles to each other; and so are the directions of the second and fourth; and the second is inclined at an angle of 60° to the first. Find the magnitude and direction of the resultant.

5. A cord PAQ, fixed to a point A, is drawn in directions AP, AQ by two forces equivalent to 3 and 4. Find what the angle PAQ must be in order that the pressure on A may be equal to 6.

6. Two forces, 2 and 3, are inclined to each other at an angle of 45°. Find their resultant.

7. If two equal forces sustain each other by means of a string passing over a tack; show that either force is to the pressure as 1 to $2 \cos \frac{1}{2}$ the angle at which the forces are inclined to each other.

8. If two forces acting on a point be in the ratio of 2 to 3; find the angle between them when their resultant is a mean proportional between them.

9. If two forces be inclined to each other at an angle of 135°; find the ratio between them when the resultant equals the less.

10. If R be the resultant of the forces P and Q acting in the

same plane on a point A, and r, p, q the respective distances of A from the perpendiculars drawn from any point in the plane on the direction of R, P, and Q respectively, then shall $Rr = Pp + Qq$.

11. Two forces, which are to each other as 2 to $\sqrt{3}$, when compounded produce a force equivalent to half the greater. Find the angle at which they are inclined to one another.

12. Three forces act perpendicularly to the sides of a triangle at the middle points, and each is proportional to the side on which it acts. Show that they will keep each other at rest.

13. If forces, proportional to the sides of any polygon, act perpendicularly to these sides respectively at their middle points, they will keep each other at rest.

14. Show that, *cæteris paribus*, the larger a carriage-wheel is, the less is the force requisite to draw the carriage over a given obstacle.

15. Two forces, 3 and 4, act on the ends of a rigid rod of length 10 feet, and are inclined to it at angles of 30° and 60° respectively. Find the magnitude and position of a force which acting on the rod shall keep it at rest.

When a body is supported by means of a string, the tension (T) of the string must be considered as one of the forces that preserve equilibrium. In the same string the tension is the same at every point and in both directions. When one string only supports a weight, the tension of the string equals the weight.

16. A given weight W, suspended from a fixed point, is drawn aside from the vertical by a horizontal force P. Find the inclination of the string to the horizon.

17. Two equal weights P are connected by a string passing over two fixed pulleys A and B, situated in a horizontal line, and support a weight W $(=\sqrt{3}P)$ hanging from a ring C, which slides freely on the string AB. Find the position of equilibrium.

18. A weight W is suspended from one extremity of a string, which passes through a ring C at its other extremity; the string passing over two fixed pulleys, A and B, in the same horizontal line. Find the position of equilibrium.

19. A and B are two given points in a horizontal line; to A a string AC is fastened $= \frac{1}{2}$ AB; to B another string is fastened passing through a ring at C, and supporting a weight. Find the position of equilibrium.

20. A circular hoop is supported in a horizontal position; and three weights of 4, 5 and 6 pounds respectively are suspended over its circumference by three strings knotted together at the centre of the hoop. Find the position of equilibrium.

21. A body of a known weight is suspended from a given point

of a horizontal plane by a string of known length, which is thrust from its vertical position by a rod (without weight) acting from a given point in the horizontal plane. Show that the tension of the string varies inversely as the tangent of inclination of the rod to the horizon.

In the last question, the thrust of the rod, acting in direction of the rod, is one of the forces keeping the body at rest. And the forces should be resolved in a direction perpendicular to the rod, in order that this unknown force may not appear.

22. Two equal weights are suspended by a string passing freely over three tacks, which form an isosceles triangle, whose base is horizontal, and vertical angle $=120°$. Find the pressure on each of the tacks.

23. If a string ACDB be 21 inches long; C and D two points in it, such that $AC=6$, $CD=7$; and if the extremities A and B be fastened to two fixed points in the same horizontal line at a distance of 14 inches from each other; what must be the ratio of two weights which hung at C and D will keep CD horizontal?

When a body is at rest, while lying on a plane, the reaction R of the plane perpendicular to the plane is one of the forces keeping the body at rest. When the body rests on a convex or concave surface, R acts in a direction perpendicular to the tangent plane at the point. In general, when R is not known or required, the forces may conveniently be resolved perpendicularly to it, in order that R may disappear.

24. A weight is supported on an inclined plane by three forces, each $= \frac{1}{2}$ the given weight, which act one vertically upwards, another horizontally, and the third parallel to the plane. Find the inclination of the plane to the horizon.

25. Two weights, P and Q, support each other on two planes, inclined to the horizon at angles of α and β respectively, by means of a string passing over the common vertex of the planes. Find the ratio of P to Q.

26. O is the centre, OA a horizontal radius, of a quadrant; over a pulley at A a string is passed supporting two weights P and 2P at its extremities, the former of which hangs at the end of the string AP, while the latter rests on the arc of the quadrant. Find the position of equilibrium.

27. Over a vertical semicircle ABD, whose centre is C, a string is laid, which is equal in length to the arc of a quadrant of the circle; and which has two weights, P and Q, at its extremities. Find the angle PCA, when the position is one of equilibrium.

28. A string passes over a pulley at the focus of a parabola, whose

axis is vertical; from one extremity of the string the weight P hangs; and at the other extremity the weight Q rests on the convex curve of the parabola. Show that P=Q; and that equilibrium exists in all positions.

29. A sphere, of weight W, rests on two planes, inclined at angles α and β to the horizon. Find the normal pressures on the planes.

In the last question, the weight of the body, supposed to be collected at its centre of gravity, is one of the forces, and acts vertically downwards.

FORCES ACTING IN ONE PLANE, BUT NOT THROUGH THE SAME POINT.

The three conditions of equilibrium are,—

1. The sum of the resolved forces in *any* direction =0.
2. Their sum in a direction at right angles with the first direction =0.
3. The sum of the moments about *any* point =0.

In determining these directions, and the position of this point, the solution is in most cases rendered more simple by resolving the forces in a direction perpendicular to the direction of some force that is not known or required, in order that this force may disappear from the equation, and by taking the moments of the forces about a point through which this unknown force passes, in order that it may disappear in another equation. And thus two equations will be all that are required, instead of three, which must otherwise be employed.

The tensions of strings, reactions of planes, weights of bodies, &c., are considered as forces. See at questions 15, 21, 23, and 29.

30. If two parallel forces act in the same direction along the opposite sides AB, DC of a parallelogram, and another force act along the diagonal BD; and if these three forces be respectively proportional to AB, DC, and BD. Find the magnitude and position of a fourth force which will keep the parallelogram at rest.

31. ABC is an isosceles triangle, C being a right angle; and three equal forces act in the lines AB, BC, CA. Show that their resultant is to one of the forces as $\sqrt{2}-1:1$; and that if CD be drawn perpendicular to AB, and DC produced to E, so that $DE=CE\sqrt{2}$; then the resultant acts through E in a direction parallel to BA.

32. An uniform beam rests on two planes inclined at angles α and β to the horizon; find the inclination (θ) of the beam to the horizon.

33. A string, having its extremities fixed to the ends of an uni-

form rod, of weight W, passes over 4 tacks, so as to form a regular hexagon; the rod (which is horizontal) being one of the sides; find the tension of the string and the vertical pressure on each tack.

34. An uniform beam PQ hangs by two strings AP, BQ from any two fixed points A and B; show that when there is equilibrium, the tensions of the strings are inversely as the sines of the angles at P and Q.

35. AD is horizontal, and DC vertical; Q is a weight hanging at one extremity of a string, which passing over a pulley at C is attached at its other extremity B to the end of an uniform beam AB; so that CB is vertical. Find the conditions of equilibrium.

Take moments about the lower end of the beam, so that the reaction may disappear.

36. An uniform beam AB hangs by a string BC from a fixed point C, with its lower extremity A resting on a smooth horizontal plane; show that, when there is equilibrium, CB must be vertical.

Resolve horizontally, so that the weight and the reaction may both disappear.

37. An uniform beam AB rests with one end A on a smooth vertical wall; the other end B is supported by a string fastened to a point C in the wall. If the length of the beam be 2 feet, and the length of the string 6 feet; find CA.

Resolve at right angles to the string, so that the tension may disappear.

38. An uniform beam AB is placed with one end A inside an hemispherical bowl, and with a point P in it resting on the edge of the bowl. If AB = 3 times the radius; find AP.

Resolve horizontally, and take moments about the centre of gravity of the beam (its middle point) so that its weight may disappear in two equations.

39. A beam AB, of weight W, rests with one end A on a horizontal plane AC, and the other end on an inclined plane CB, whose angle of inclination to the horizon is 60°. If a string CA (=CB) prevent the beam from sliding, what is the tension of this string?

Resolve horizontally, and take moments about A, so as to get rid of the reaction at A.

40. A beam AB, whose weight is W, and length 6 feet, rests on a vertical prop CD(=3 feet); the other end A is on a horizontal plane, and is prevented from sliding by a string DA(=4 feet). Find the tension of this string.

Resolve horizontally, and take moments about A.

41. Two beams, CA and CB, whose weights are proportional to their lengths, rest against each other on a smooth horizontal plane

DE; the other ends A and B resting against two smooth vertical walls DA, and EB. Find the equations for determining the angles ACD, and BCE.

42. A beam, of weight W, is placed with one end on a vertical, and the other on a horizontal plane, and is kept at rest by a horizontal force acting at the lower end; find (1) this horizontal force; (2) the pressure on the vertical plane; (3) the pressure on the horizontal plane.

Resolve horizontally and vertically, and take the moments about the lower end.

43. A beam AB, has two strings fastened to its ends, one of which, AC(= length of beam), is fastened to a ring C; *through* which ring the other string BCP passes, supporting a weight $=\frac{1}{2}$ the weight of the beam. Find the inclination of the beam to the horizon.

Resolve perpendicularly to AC, and take moments about A, in order that the unknown tension of the string AC may disappear.

44. Three uniform beams, AB, BC, CD, of the same thickness, and of lengths l, $2l$, and l respectively, are connected by hinges at B and C, and rest on a perfectly smooth sphere, the radius of which $=2l$; so that the middle point of BC, and the extremities A and D are in contact with the sphere. Find the pressure at the middle point of BC.

Resolve the 6 forces vertically. And then, considering AB kept at rest by two forces about a fixed point B, take the moments of these forces about this point.

45. A roof ACB consists of beams which form an isosceles triangle, of which the base AB is horizontal. Given W the weight of each beam, and α the angle at which it is inclined to the horizon; find the force necessary to counterbalance the horizontal thrust at A.

Resolve horizontally, and take moments about A, so that the unknown reaction at A may disappear.

46. A string ACB, having two equal weights attached to its extremities A and B, passes over a pulley C, and over two pegs D and E, situated at equal distances from the vertical line through C. A smooth ring PQ, whose diameter = diameter of the pulley, and whose weight = each of the weights at A and B, is passed over the string so as to be between the pulley and the pegs. When there is equilibrium, find the inclination of DP to the horizon.

Consider the ring to be supported in its position by 5 forces, viz. the tensions of the 4 strings PD, PC, QE, QC, and its own weight; and resolve vertically.

47. A smooth sphere, of radius 9 inches, and weight 4 lbs., is kept at rest on a smooth plane, inclined at an angle of 30° to the

horizon, by means of an uniform beam, of length 7 feet, moveable about a hinge on the plane, and resting on the sphere. Find what must be the weight of the beam, that it may be inclined at an angle of 15° to the plane.

> Consider the sphere kept at rest by 3 forces; its own weight, and the reactions of the plane and of the beam. Resolve parallel to the plane, so that one of these reactions may disappear. Then consider the beam as kept at rest round a fixed point by two forces; its own weight, and the reaction of the sphere.

48. A beam, of length $2a$, rests with one end leaning against a smooth vertical wall, and is supported by a prop whose perpendicular distance from the wall is b. Find the inclination of the beam to the horizon.

> Resolve vertically, and take the moments about the point at which the beam touches the wall, in order that the reaction at that point may disappear.

49. An uniform beam, 6 feet in length, rests with one end against a smooth vertical wall, the other end resting on a smooth horizontal plane, and is prevented from slipping by a horizontal force, applied at that end, equal to the weight of the beam; and by a weight $=\frac{2}{3}$ the weight of the beam, suspended from a certain point on the beam. Find the distance of this point from the lower end of the beam, if the beam be inclined at an angle of 45° to the horizon.

> Resolve horizontally, and take moments about the lower end of the beam, so that the reaction of the horizontal plane may disappear.

50. A rod, of length $2a$, rests with one end on the concave surface of a paraboloid inverted, and is supported by a prop at its focus. If $4m$ be the latus rectum; find θ the inclination of the rod to the horizon, and x the distance of the lower end of the rod from the focus. Show also that, if the length of the rod be greater than the latus rectum, the beam will only rest in a vertical position.

> Resolve in direction of a tangent at the point where the rod rests on the surface, and take moments about that point, so that the reaction may disappear.

51. An uniform lever, whose arms, of lengths $2a$ and $2b$, are at right angles to each other, touches the circumference of a circle, whose plane is vertical, and radius $=c$. Find the inclination of the arm $2a$ to the horizon.

> Resolve in direction of one arm, and take moments about the point of contact of the other, so that one of the reactions may disappear.

52. A given weight W is to be supported by a horizontal rod, without weight, and of length b, on two vertical props, one of which can sustain no more than P, and the other no more than Q. Required the distance, from each prop, of the point at which W must be suspended, in order that the pressure on each prop may be less by the same quantity than the greatest pressure they can support respectively.

If the body have a *fixed* point, the only necessary condition of equilibrium is that

The sum of the moments of all the forces about that point $=0$.

53. A bar weighs a ounces per inch. Find its length when a given weight na, suspended at one end, keeps it in equilibrium about a fulcrum at a distance of b inches from the other end.

54. Two weights are suspended from the arms of a bent lever without weight, which are inclined to the horizon at angles of 30° and 45° respectively; the first arm being 18 inches and the second 12 inches long. Find the proportion of the weights.

55. On a straight lever, weighing 6 lbs., and of 6 feet in length, weights of 1, 2, 3, 4, 5 lbs. are hung at respective distances of 1, 2, 3, 4, 5 feet from the extremity. Required the position of the fulcrum, about which the whole will rest.

56. A beam, 30 feet long, balances itself on a point at one-third of its length from the thicker end; but when a weight of 10 lbs. is suspended from the smaller end, the prop must be moved 2 feet towards it, in order to maintain the equilibrium. Find the weight of the beam.

57. An uniform bent lever ACB hangs freely by one extremity A. If C be a right angle, $AC=2a$, $BC=2b$; find the inclinations of AC to the horizon.

58. One end of a beam is connected with a horizontal plane by means of a hinge, about which the beam can revolve in a vertical plane; the other end is attached to a weight ($=3$ times weight of beam) by means of a string passing over a pulley in a vertical wall. If the length of the beam $=$ the distance of the hinge from the wall $=$ the height of the pulley above the plane; find the inclination of the beam (θ), and of the string (ϕ) to the horizon.

59. An uniform beam AB is moveable in a vertical plane about a hinge at A; to the other end B a string is attached, which passing over a fixed pulley at C ($AC=AB$) supports a weight $=$ half the weight of the beam. Find the inclination of the beam to the horizon.

60. A heavy rod AB ($=a$) is moveable in a vertical plane about a hinge at A, and supports with its other extremity B another heavy rod CD ($=b$), moveable in the same plane about a hinge at C. If

AC ($=c$) be horizontal, what must be the ratio between the weights of the two beams that CB may equal AB?

Consider separately the forces acting on the two beams.

61. AC, CB are equal arms of a straight lever, whose fulcrum is C; to C a heavy arm CD is fixed, at right angles to AB. Prove that, when different weights are suspended from the extremity A, the tangents of the inclinations of AB to the horizon will be proportional to the weights.

62. Four weights, 1, 3, 7 and 5, are at equal distances on a straight lever without weight. Where must be the fulcrum on which they balance?

63. P and Q are weights fixed to the extremities of a circular arc (whose chord $=2a$, and height $=b$), and which is placed with its plane vertical on a plane inclined at an angle α to the horizon. Find the ratio of P to Q, in order that the arc (prevented from sliding) may rest with its chord parallel to the plane.

Suppose the point of contact of the arc and plane to be fixed, and take moments of P and Q about that point.

64. A straight uniform rod AC, of 12 lbs. weight, and moveable in a vertical plane about a hinge at C, has two equal weights of 2 lbs. each, suspended from the extremity A, and from the middle point B; and is kept at rest by a string attached at A, passing over a fixed pulley D, and supporting a weight of 6 lbs. If CD be horizontal, and equal to CA; find the inclination of the rod to the horizon.

65. An uniform bent lever, when supported at the angle, rests with the shorter arm horizontal; but if the shorter arm were twice as long, it would rest with the other arm horizontal. Find the ratio between the lengths of the arms and the angle at which they are inclined to each other.

66. At what point of a tree must a rope of given length a be fixed, so that a man pulling at the other end may exert the greatest force in upsetting it?

Find the greatest moment about the foot of the tree.

67. If a piece of timber, 17 feet long, be placed on a prop 4 feet from one end, it is found that a hundred-weight at that end would be balanced by 12 lbs. at the other; but that, if the places of the weights be exchanged, the prop must be 8 feet from the other end. Find the weight of the timber, and the position of the prop, on which it would balance without any weights.

That is, find the point at which the collected weight may be supposed to act, or the centre of gravity; because when this point is supported, the whole beam will be at rest.

68. If $n+1$ bodies, P, 2P, 3P, 4P, &c. be placed at equal distances along a straight rod without weight, and of length na; find the point on which the whole will balance.

69. An uniform beam, 18 feet long, rests in equilibrium upon a fulcrum 2 feet from one end; having a weight of 5 lbs. at the further, and one of 110 lbs. at the nearer end to the fulcrum. Find the weight of the beam.

70. AC and BC are two uniform rods of equal lengths, and perpendicular to each other in a vertical plane; but the weight of BC : that of AC $=\sqrt{3}:1$. At what angle will BC be inclined to the horizon, when the angular point C rests on a horizontal plane, and the whole is kept in equilibrium?

FORCES WHICH DO NOT ACT IN THE SAME PLANE.

The following are the 6 conditions of equilibrium of any forces, acting not in the same plane, on a free rigid body.

Take *any* 3 lines mutually at right angles to each other, which call the axes of x, y and z; then

The sums of the resolved forces in the directions of the axes of x, y and z are *separately* $=0$.

And the sums of the moments of the forces about the axes of x, y and z are *separately* $=0$.

71. A right-angled triangle, whose sides are 3, 4 and 5, without weight, rests horizontally on three props placed at its angular points. Find the distances of a point in its plane from the sides containing the right angle, on which if a weight be placed, the pressure at each prop may be proportional to the opposite side.

Take the sides containing the right angle for the axes of x and y. Resolve parallel to axis of z, and take moments about axes of x and y.

72. Any triangle is supported at its angular points, and a weight is laid on it at its centre of gravity. Show that the pressures at the three props are equal.

73. A heavy triangle, of uniform thickness and density, is supported in any position by three vertical strings fastened to the angular points. Show that each string supports an equal portion of the weight.

Take the moments about one of the sides, considered as one of the coordinate axes.

74. A given weight W is supported by n strings passing over pulleys placed at the angular points of a regular polygon of n sides whose plane is horizontal, each string being fastened to an equal weight P. Find the inclination (θ) of each string to the vertical. Show that W must be less than nP; and that the strings can never become horizontal, unless P is infinitely great compared with W.

THE CENTRE OF GRAVITY.

The centre of gravity of a triangle is at the point where the lines, joining the middle points of the sides with the opposite angles, intersect.

Of a parallelogram, at the intersection of the diagonals.

Of a circle, or of an ellipse, or of any regular figure, at the centre.

Of a semicircle; in the axis, at a distance from the centre $=\frac{\text{diameter}}{3\pi}$.

Of a cylinder or prism; at the middle point of the axis joining the centres of gravity of the two ends.

Of a cone or pyramid; in the line joining the vertex with the centre of gravity of the base, at a distance from the vertex $=\frac{3}{4}$ of the whole length of the line.

Of a sphere or spheroid; at the centre.

Of a hemisphere; in the axis, at a distance from the vertex $=\frac{5}{8}$ of radius.

Of a hemispherical surface; at the middle point of the axis.

Of a paraboloid; in the axis, at a distance from the vertex $=\frac{2}{3}$ of the axis.

75. If G be the centre of gravity of the triangle ABC, then $3(GA^2+GB^2+GC^2)=AB^2+AC^2+BC^2$.

If a body (or system of particles) be made up of component bodies (or particles), then the moment of the whole, supposed to be collected at its centre of gravity, about any line or plane = the sum of such moments of the component bodies (or particles).

76. Two arms of a bent lever are a and b, and the angle between them is α; find the distance of the centre of gravity of the whole from the angular point.

77. Find the perpendicular distance of the centre of gravity of any quadrilateral figure from either of the diagonals.

78. Find the distance of the centre of gravity of the frustum of a cone from the base; a and b being the radii of the two ends, and c the altitude of the frustum.

Complete the cone; and find the distances of the centres of gravity of the large and of the small cone from the base of the frustum.

79. If two cones have the same base, and their vertices towards the same parts; find the distance of the centre of gravity of the solid contained between their two surfaces from their common base.

80. Find the distance of the centre of gravity of the frustum of

any pyramid from the base, a and b being any two homologous sides of the ends, and c the altitude of the frustum.

81. Find the distance of the centre of gravity of the frustum of a paraboloid from the base, a and b being the radii of the two ends, and m the latus rectum.

82. If two spheres of radii a and b, touch each other internally; find the distance of the centre of gravity of the solid contained between the two surfaces, from the point of contact.

83. Find the distance of the centre of gravity of a hemispherical bowl from the base, a being the internal radius, and c the thickness. Thence find the position of the centre of gravity of a hemispherical surface.

> When a body is suspended by one point, the straight line joining the point of suspension and the centre of gravity is vertical.

84. If a triangle, whose sides are 3, 4 and 5, be suspended from the centre of the inscribed circle; find in what position it will rest.

> When a body, acted on by gravity only, is in equilibrium with one point of it resting on a surface, the line joining this point of contact with the centre of gravity is vertical.
>
> Therefore, if the body be a solid of revolution, the centre of gravity is always at the intersection of the axis, and the normal at the point of contact.

85. A paraboloid of revolution, whose axis $=a$, and radius of base $=b$, rests with its convex surface on a horizontal plane. Find θ the angle of inclination of its axis to the horizon.

86. A solid is composed of a cone and a hemisphere, of equal bases, placed base to base. Find the ratio between the dimensions of the cone and hemisphere, in order that the whole may be at rest with *any* point of the spherical surface on a horizontal plane.

87. A solid is composed of a hemisphere and a paraboloid, of equal bases, placed base to base. Find the ratio between their dimensions, in order that the whole may be at rest with *any* point of the spherical surface on a horizontal plane.

> The volume of the paraboloid $=\frac{1}{2}$ the volume of the circumscribing cylinder.

> If a body rest with its base on a plane, it will fall over or not according as the vertical through the centre of gravity falls without or within the base.

88. ABCD is a quadrilateral figure of which the two shorter sides AB, BC are equal, as also the two longer sides AD, DC; and the angle ABC is a right angle. If the length of AB be given, what is

the greatest length of AD, that the figure may rest with the base AB on a horizontal plane, without oversetting?

89. What is the least angle of inclination that a plane must have to the horizon, for a prism on a regular base of n sides to roll down it?

90. A cone is placed with its base on a plane inclined at an angle of 30° to the horizon, and is prevented from sliding. Find the smallest vertical angle it can have that it may not fall over.

91. A paraboloid, of given parameter m, when prevented from sliding down a plane whose inclination is α, just stands without falling over. Find the length of the axis of the paraboloid.

Guldinus's properties are,—

1st. The surface generated by a curve line, revolving about a fixed axis = the product of the length of the curve into the length of the path described by its centre of gravity.

2nd. The volume generated by a plane area revolving about a fixed axis in its own plane = the product of the area into the length of the path described by its centre of gravity.

92. Find the volume generated by the revolution of a right-angled triangle, whose sides are a and b, about the hypothenuse.

93. Determine the distance of the centre of gravity of a semicircular area from its base.

94. A parallelogram and a triangle, on the same base and between the same parallels, revolve around the base as an axis. Compare the solids they generate.

95. The distance of the centre of gravity of a cycloid from the vertex $=\frac{7}{12}$ of the axis. From this property, compare the volumes generated by the revolution of the cycloid, 1st about the base, and 2nd about a tangent at the vertex.

96. From the expression for the centre of gravity of a trapezoid, determine the volume of the frustum of a cone, the radii of whose ends are b and c, and the altitude a.

MACHINES.

1. On the straight lever, where a and b are the arms, at the extremities of which P and W respectively act,

$$P.a=W.b.$$

2. On the bent lever, the arms being inclined at angles of α and β respectively to the horizon,

$$P.a\cos\alpha=W.b\cos\beta.$$

3. On the Roman steelyard, where the fulcrum C is fixed, and the weight P constant; if W be the weight to be deter-

mined, hanging from the fixed point B; P the given weight, hanging from the variable point M; G the centre of gravity of the beam whose weight is w,

$$W \times CB = P \times CM + w \times CG.$$

4. On the Danish steelyard, where the fulcrum C is moveable; w the weight of the beam, G its centre of gravity; W the weight to be determined, hanging from the fixed point B,

$$W \times BC = w \times GC.$$

5. On the wheel and axle,

$$P : W = \text{radius of axle} : \text{radius of wheel}.$$

6. On the single moveable pulley, the strings being parallel,

$$2P = W.$$

7. On the same, if the strings be inclined to each other at an angle of 2α,

$$2P \cos\alpha = W.$$

8. On the first system of pulleys, each pulley hanging by a separate string, if n be the number of *moveable* pulleys,

$$2^n P = W.$$

9. On the second system of pulleys; the same string passing round all the pulleys; if n be the number of strings at the lower block,

$$nP = W.$$

10. On the third system of pulleys; each string being attached to the weight; if n be the number of pulleys (including that which is fixed),

$$(2^n - 1)P = W.$$

11. On the inclined plane; if α be the inclination of the plane to the horizon; β the angle which the direction of P makes with the inclined plane,

$$P \cos\beta = W \sin\alpha.$$

12. On the screw,

$P : W =$ distance between two threads : circumference described by the point of application of P.

In the following problems no account is taken of friction.

97. There are $n+1$ weights, a, b, c, &c., in geometrical progression, and a placed at one extremity A of a lever balances b placed at the other extremity B. Prove that the first n weights placed at A will balance the last n weights placed at B.

98. In rowing, if the oar be 12 feet long, and the rowlock $2\frac{1}{2}$ feet from the handle, compare the pull of the rower with the force exerted against the water by the boat.

99. If the common balance consist of a straight uniform beam AB, whose length is $2a$, and weight W; the centre of gravity of the whole being at G, the middle point; but the point of suspen-

sion at C, at a distance b directly above G; and if unequal weights P and Q be placed at A and B respectively; find at what inclination (θ) to the horizon the beam will rest.

100. An apparent pound weight is weighed at each end of a false balance; and it is found that the sum of the real weight added to what are the apparent weights at each extremity $= 2\frac{1}{2}$ lbs. Find the ratio between the lengths of the arms.

101. A false balance has one of its arms exceeding the other by $\frac{1}{m}$th part of the shorter. Supposing a shop-keeper, in using it, puts the weight as often in one scale as the other; what is his gain per cent. on what is sold?

102. The whole length of the beam of a false balance is 3 ft. 9 in.; a certain body, placed in one scale, appears to weigh 9 lbs.; and placed in the other, appears to weigh 4 lbs. Find the true weight of the body, and the lengths of the arms of the balance.

103. The arms of a bent lever are 3 feet and 5 feet, and inclined to each other at an angle of 150°; and at their extremities weights of 7 lbs. and 6 lbs. respectively are suspended. Find the inclination of each arm to the horizon, when there is equilibrium.

104. The lengths of the arms, their inclination to each other, and the weight at the extremity of the shorter arm being the same as in the last question; find what the other weight must be, in order that, 1st the shorter, and 2nd the longer arm may rest in a horizontal position.

105. A Roman steelyard, whose weight is 10 lbs., has its centre of gravity at a distance of 2 inches from the fulcrum; and the weight to be determined is supported by a pan placed at a distance of 3 inches on the other side. Find the respective distances from the fulcrum at which the constant weight of 5 lbs. must be placed, in order to balance 10, 20, 30, &c. lbs. placed successively in the pan.

106. The distance of the centre of gravity of a Danish steelyard from the extremity (where the pan containing the weight to be determined is fixed) is 18 inches; and the weight of the beam is 3 lbs. Find at what distances, respectively, from the same extremity the fulcrum must be placed, when weights of 4, 8, 12, 16, 20, &c. ounces are placed successively in the pan.

---

107. If a power of 10 lbs. balance a weight of 555 lbs. on a wheel whose diameter is 4 yards; what must be the radius of the axle? The thickness of the rope being neglected.

108. The radius of the wheel being 2 feet, and of the axle 5 inches, and the thickness of the rope being $\frac{3}{4}$ inch; find what power

will balance a weight of 130 lbs. The power being supposed to act in the axis of the rope.

109. The radius of a wheel being 3 feet, and of the axle 3 inches; find what weight will be supported by a power of 120 lbs. The thickness of the rope coiled round the axle being 1 inch.

110. There are two wheels, whose respective diameters are 5 feet and 4 feet, on the same axle; the diameter of the axle being 20 inches. What weight on the axle would be supported by forces equal to 48 lbs. and 50 lbs. on the larger and smaller wheels respectively?

---

111. At what angle must the strings of a single moveable pulley be inclined to each other, in order that P may equal W?

112. How many pulleys (supposed to be without weight) must there be in the first system, in order that 1 lb. may support a weight of 128 lbs.?

113. In the first system there will be equilibrium, if the power, the weight, and each pulley are all equally heavy.

114. In the first system of 6 moveable pulleys, find the ratio that the weight of each pulley must bear to the power, in order that the latter may just be balanced by the weight of the pulleys alone.

115. In the second system of pulleys, if there be 10 strings at the lower block; find what power will support a weight of 1000 lbs.

116. In the third system of 6 pulleys (supposed to be without weight); find what weight will be supported by a power of 12 lbs.

117. In the third system of 8 pulleys, find the ratio that the weight of each pulley must bear to the weight supported, in order that the latter may just be supported by the weight of the pulleys alone.

---

118. What force is necessary to support a weight of 50 lbs. on a plane inclined at an angle of 15° to the horizon; the force acting horizontally?

119. If, on an inclined plane, the pressure, force and weight be as the numbers 4, 5, and 7; find the inclination of the plane to the horizon, and the inclination of the force's direction to the plane.

120. If the weight, power, and pressure on an inclined plane be respectively as the numbers 25, 16, and 10; find the inclination of the plane, and the inclination of the power's direction with the plane.

121. A weight W is just supported on an inclined plane by a force P, acting by means of a wheel and axle placed at the top, so that the string attached to the weight is parallel to the plane. Given R and r the radii of the wheel and of the axle; find the plane's inclination to the horizon.

---

122. What force must be exerted to sustain a ton weight on a screw, the thread of which makes 150 turns in the height of 12 inches; the length of the arm being 6 feet?

123. Find the weight that can be sustained by a power of 1 lb., acting at the distance of 3 yards from the axis of the screw; the distance between two contiguous threads being 1 inch.

124. What must be the length of a lever, at whose extremity a force of 1 lb. will support a weight of 1000 lbs. on a screw; the distance between two contiguous threads being $\frac{3}{4}$ inch?

FRICTION.

When a body rests on any surface, the friction $=\mu R$, where R is the normal pressure at that point; μ being constant and called the coefficient of friction.

This friction is therefore to be accounted for as one of the forces acting on the body, and is considered to act in a direction opposite to that in which the body is on the point of moving. But, as it is a force of a peculiar nature, not causing motion, but only preventing other forces from causing motion, there may be a great many conditions of equilibrium, with either of which, the friction is sufficient to present sliding. But there will be, in general, limits to those conditions, with one of which, motion is supposed to be on the point of taking place in one direction, and with the other, in the opposite direction. One of these conditions may always be found from the other, by changing the sign of μ; which is equivalent to reckoning the friction in the opposite direction.

The coefficient of friction is, for practical purposes, often expressed in terms of the inclination of the plane, on which the body being placed will be just on the point of descending. The friction μR is thus a force acting *up* the plane, which keeps the body at rest, and therefore $\mu R : R = \tan \alpha : 1$, or $\mu = \tan \alpha$, α being the plane's inclination.

125. A given weight W is sustained on a rough plane, whose angle of inclination to the horizon is α, by a power P, inclined at an angle β to the plane; μ being the coefficient of friction. Find between what limits P must lie.

126. Determine the least force which will drag a weight of 50 lbs. along a rough horizontal plane, the friction being such as would just prevent the body from sliding down a plane of inclination 30°. Find also at what angle this least force must be inclined to the horizon.

127. A body is supported on a rough plane by a force equal to its own weight, and is just on the point of sliding down the plane. Required the sum of the angles at which the plane is inclined to the horizon and to the force's direction.

128. An isosceles triangle, whose base is to one of its equal sides as $1 : \sqrt{7}$, is placed with its base on an inclined plane; and it is found that when the body begins to slide, it also begins to roll over. Find the coefficient of friction.

129. An uniform rectangular beam AB, of weight W, is supported against a rough wall CD by a string AC passing over a pulley at C, to which a weight P is attached. If the inclination of the plane which measures the friction be 30°; and if the centre of gravity of the beam be at G the middle point of ADB; find whether P must be greater or less than W.

Resolve vertically, and horizontally, and take moments about D.

130. An uniform beam, of weight W, leans against a vertical wall, and has its lower end resting on a horizontal plane. If μ and μ' be the coefficients of friction of the wall and of the plane respectively; find the value of θ the inclination of the beam to the horizon, when motion is just on the point of taking place. Find also the pressures on the wall and on the plane.

Resolve vertically, and horizontally, and take moments about the lower end of the beam.

131. A ladder rests against a vertical wall, to which it is inclined at an angle of 45°; the coefficients of friction of the wall and of the horizontal plane being respectively $\frac{1}{3}$ and $\frac{1}{2}$; and the centre of gravity of the ladder being at its middle round. A man, whose weight = half the weight of the ladder, ascends it; find to what height he will go before the ladder begins to slide.

Resolve, and take moments as in the last example.

132. A hemisphere rests between a vertical wall and a horizontal plane; μ and μ' being the coefficients of friction of the wall and plane respectively. Find θ the inclination of the base of the hemisphere to the horizontal plane, in the limiting position of equilibrium.

Resolve vertically, and horizontally, and take moments about the centre of the sphere.

DYNAMICS.

1. THE COLLISION OR IMPACT OF BODIES.

General Formulæ. Let A, B be the masses of the two bodies that impinge, one on the other; a, b their velocities before impact and measured in the same direction; u, v the velocities after impact; ϵ the common elasticity of the two bodies: then will

$$u=\frac{Aa+Bb-\epsilon B(a-b)}{A+B},$$
$$v=\frac{Aa+Bb+\epsilon A(a-b)}{A+B}.$$

N.B. When the bodies are inelastic, $\epsilon=0$; when perfectly elastic, $\epsilon=1$.

1. A and B weigh 12 lbs. and 7 lbs. respectively, and move in the same direction with velocities of 8 feet and 5 feet in a second; find the common velocity after impact; also the velocities lost by A and gained by B respectively.

2. A, moving with a velocity (11) impinges upon B moving in the opposite direction with a velocity (5); and by the collision A loses one-third of its momentum; what are the relative magnitudes of A and B?

3. A, weighing 8 lbs., impinges upon B weighing 5 lbs. and moving in A's direction with a velocity of 9 feet in 1″; by collision B's velocity is trebled; what was A's velocity before impact?

4. The centres of two balls, A and B, move along the same straight line with velocities a and b. Find the velocity of each after impact when $6A=5B$, a is 7 feet in 1″, $4a+5b=0$, and $\epsilon=\frac{2}{3}$.

5. A and B are two perfectly hard balls meeting in opposite directions; A is three times as large as B, but moves with a velocity of 12 feet in 1″, which is only $\frac{2}{3}$rds of B's velocity; what is the common velocity after impact?

6. A : B as 3 : 2, and the velocity of A : velocity of B as 5 : 4; they are perfectly hard bodies and move before impact in the same direction. What is the proportion of A's velocity lost by A and gained by B?

7. A and B are perfectly elastic and in the ratio of 4 : 3; they are moving in the same direction with velocities as 5 : 4; what is the ratio of the velocities of A and B after impact?

8. A and B are perfectly elastic; they are moving in opposite directions; A is treble of B, but B's velocity is double that of A; determine the motions after impact.

9. There is a row of perfectly elastic bodies in an increasing geometrical progression whose common ratio is 3, and placed contiguous to each other; the first impinges upon the second, which transmits its velocity to the third, and so on; the last body moves off with $\frac{1}{64}$th of the velocity of the first body; what was the number of bodies?

10. $A(=3B)$ impinges upon B at rest; A's velocity after impact is $\frac{3}{5}$ths of its velocity before impact; required the value of ϵ which measures the elasticity.

11. At what angle must a body, whose elasticity is $\frac{1}{3}$, be incident on a perfectly hard plane that the angle between the directions before impact and after, may be a right angle?

12. A and B are two balls of given elasticity; what must be the magnitude of a third ball, that the velocity communicated from A to B by the intervention of this ball may equal that communicated immediately from A to B? Determine also the limits within which this problem is possible.

13. If A communicate velocity to B through a number of other bodies which are geometric means between A and B, find the limit to which the velocity of B will continually approach when the number of means is continually increased, the bodies being perfectly elastic.

14. ABD is a semicircle, AB the diameter, DC the sine of BD. An inelastic body moving uniformly along AD impinges on the plane CD; prove that it will move uniformly along DC, and that the time along DC will vary as tan BD.

15. A ball, whose elasticity is ϵ, projected from a given point in the circumference of a circle, after being reflected from it twice returns to the given point. Required the direction of projection.

16. The sides AB, CD of a billiard table are parallel; and an imperfectly elastic ball struck from a point C in one side, impinges at E in the other, and is reflected to D in the first side. Show that the time along CE : time along ED :: ϵ : 1.

17. A row of four balls A, B, C, D of perfect elasticity is placed in a straight line. Required the ratio of their masses, that the momentum of A may, after impact, be equally divided among the four; B, C, D being originally at rest.

Find the result also when there are n balls.

18. PQ is a given vertical line terminating in a hard horizontal plane at Q; a perfectly elastic ball being dropped from P meets another perfectly elastic ball rebounding with a known velocity from Q, and both are reflected back; to find where they must meet in order that they may thus rebound from one another continually.

19. An imperfectly elastic ball, descending vertically from rest, meets a horizontal plane, which is moving uniformly in an opposite direction; having given the distance between the plane and the body at first, and the degree of elasticity; find the velocity of the plane so that the body may return to the point from which it fell.

20. The points A, B, C, &c. are taken at the distance of four feet from each other successively in the vertical line AS. A body, whose elasticity is $\frac{1}{2}$, beginning its descent from A impinges on a perfectly hard horizontal plane at B, which is now removed, and in its second descent the body impinges on another such plane at C, which is then removed; and so on. Find the space described between the 7th and 8th rebounds, and the time of describing this space.

21. The direction and velocity of the motion of the centre of gravity of two bodies is not altered by their impact.

22. The magnitudes of three perfectly elastic bodies are in harmonical progression; prove that the momentum communicated to either of the extremes by the impact of the other, equals the momentum of the mean moving with the velocity of the impinging body before impact.

23. Two bodies, A and B, whose elasticity is $\frac{2}{3}$, moving in opposite directions with velocities 25 feet and 16 feet in $1''$ respectively, impinge directly upon each other; find the distance between them when $4''\cdot 5$ from the moment of impact have elapsed.

24. An imperfectly elastic ball moves in a horizontal plane and impinges on a hard vertical plane obliquely, its direction making an angle α with the normal to the plane; find the velocity and direction of the motion after impact.

25. A and B are two balls whose elasticity is $\frac{3}{5}$, and A strikes B at rest; prove that, if B be infinitely greater than A, A's momentum before impact : momentum communicated to B :: 5 : 8.

26. In the direct impact of perfectly hard bodies, prove that the difference of the *vires vivæ*, before and after impact, is equal to the sum of the *vires vivæ* of the bodies moving with the velocities lost and gained respectively.

Def. The '*vis viva*' expresses the product of the mass of a body into the square of its velocity.

27. Find the elasticity of two bodies A and B, and their proportion to each other, so that when A impinges upon B at rest, A may remain at rest after impact, and B move on with $\frac{1}{6}$th part of A's velocity before impact.

28. In the oblique impact of two perfectly elastic bodies, the sum of each body multiplied into the square of its velocity is the same before and after impact.

29. A ball A in motion is struck by an equal ball B moving with the same velocity, and in a direction making an angle of α° with that in which A is moving, in such a manner that the line joining their centres at the time of impact is in the direction of B's motion; find the velocities of the bodies, whose elasticity is ϵ, after impact, and show that that of A will be greatest when $\tan\frac{\alpha}{2}=\sqrt{\frac{\epsilon}{1+\epsilon-\epsilon^2}}$.

30. A ball of given elasticity ϵ is to be projected from a given point P, so that being reflected at any number of given planes in a given order, it may afterwards strike a given point Q; find the direction of projection by a geometrical construction.

UNIFORMLY ACCELERATED MOTION AND GRAVITY.

General Formulæ. Let s denote the space described by a body in the time t'', acted on by an uniformly accelerating force f. At the beginning of the t'', the body is supposed to be at rest, and at the end of t'' to have acquired a velocity v. We have then,

1. $s=\frac{1}{2}ft^2=\frac{1}{2}tv=\frac{v^2}{2f}$.

2. $v=ft=\frac{2s}{t}=\sqrt{2fs}$.

3. $t=\frac{v}{f}=\frac{2s}{v}=\sqrt{\frac{2s}{f}}$.

4. $f=\frac{v}{t}=\frac{v^2}{2s}=\frac{2s}{t^2}$.

If a body be projected with a velocity V in a direction coincident with or opposite to that in which f acts; then

5. $s=Vt+\frac{1}{2}ft^2$, or $Vt-\frac{1}{2}ft^2$ respectively.

6. $v^2=V^2+2fs$, or V^2-2fs in like manner.

If P and Q be two weights, supported on two inclined planes that have a common vertex, and whose angles of inclination to the horizon are α, β respectively; then

7. $$f = \frac{\text{moving force}}{\text{mass moved}} = \frac{Pg \sin \alpha - Qg \sin \beta}{P+Q} = \frac{P \sin \alpha - Q \sin \beta}{P+Q} g,$$
where g measures the force of gravity, and is here supposed to be equal to 32·2 feet.

31. A body has been falling for 11″; find the space described, and the velocity acquired.

32. Find the time in which a falling body would acquire a velocity of 500 feet in 1″; and the height from which it will have fallen.

33. What is the velocity acquired by a body when it strikes the earth, having been dropped from a height of 450 feet; and the momentum acquired by a body of 10 stone weight?

34. A body has been falling for 15″; compare the spaces described in the seventh and last seconds.

35. A body had fallen through a height equal to $\frac{1}{4}$ of a mile; what was the space described by it in the last second?

36. A body has been falling for 12″·5; what was the space described in the last second but 5 of its fall?

37. The space described by a body in the 5th second of its fall was to the space described in the last second but 4, as 1 to 6; what was the whole space described?

38. A body is projected vertically upwards with a velocity of 64 feet in 1″; how far will it ascend before it begins to return?

39. A body is projected vertically upwards, and the time between its leaving a given point and returning to it is given; find the velocity of projection, and the whole time of motion.

40. With what velocity must a stone be projected from the top of a tower, 250 yards above the sea, that it may reach the water in 6″?

41. Upon a steeple 150 feet high is a spire of 40 feet; at the same instant that a stone was let fall from the top of the steeple, another was projected vertically upwards from the bottom of it, with a velocity sufficient to carry it to the top of the spire; at what point will these stones meet?

42. A person ascending vertically in a balloon lets fall a stone when at a given height; find the time t of the stone's reaching the ground, supposing the velocity of the balloon at the given altitude known; and explain the meaning of the negative value of t.

43. If a body fall through a distance a at two different places, and if the time of falling at one place be T″ less, and the velocity acquired m feet greater than at the other, compare the force of gravity at the two places.

44. A body is projected upwards from the lower extremity of a vertical line, 250 feet high, with a velocity of 90 feet in 1″; after

what time must another be projected downwards from the upper extremity with the same velocity, so as to meet the former in the middle point of the line?

45. A body projected in the direction of the action of a constant force describes P and Q feet in the *p*th and *q*th seconds; find the magnitude of the force, and the velocity of projection.

46. Two bodies are let fall from the same point, at an interval of $1''$; how many feet are they apart at the end of $1'$ from the fall of the first?

---

47. The length of an inclined plane is 400 feet, its height 250; a body falls from rest from the top of the plane; what space will it have fallen through in $3''\cdot5$; what time will it be in falling through 300 feet; and what velocity will it have acquired when it has arrived within 50 feet of the bottom of the plane?

48. The elevation of a plane is 25° 30′; a body in falling from the top to the bottom of it acquires a velocity of 450 feet in $1''$; required the length of the plane.

49. The length of a plane is 240 feet, and its elevation is 36°; mark out upon it a part equal to the height which a body, falling down the plane, describes, whilst another body would descend freely through the height.

50. A body descending vertically draws an equal body 25 feet in $2\frac{1}{2}''$ up a plane inclined at 30° to the horizon by means of a string passing over a pulley at the top of the plane; determine the force of gravity.

51. Two bodies start from the top of an inclined plane, one falling down the length of the plane, and the other down its height; it is observed that the former is 3 times as long as the latter in reaching the base. Required the inclination of the plane.

52. With what weight must a given weight of 7 oz. be connected by a string passing over a single fixed pulley, so as to describe the same space in a given time as when it descends freely down an inclined plane whose inclination is 30°?

53. AC, BC are two inclined planes meeting a horizontal plane at the same point C. Having given the inclinations of the planes and the length of BC, find the point A, such that a body descending down AC may just ascend to the top of the plane BC.

54. Two equilateral triangles in the same vertical plane are placed with their bases at a distance of *d* feet from each other upon the same horizontal line, and a non-elastic body falls down the side of the first, along the space between the bases and up the side of the second triangle, the vertex of which it just reaches; given the side of the first triangle equal to *a* feet; find the side of the second, and the whole time of motion.

55. A ball having descended to the lowest point of a circle through an arc whose chord is *a*, drives an equal ball up an arc whose chord

is b; show that the common elasticity e of the two balls is equal to $2\frac{b}{a}-1$.

56. If a body be projected down a plane inclined at 30° to the horizon, with a velocity equal to $\frac{3}{4}$ths of that due to the height of the plane, the time down the plane will equal the time down its vertical height from rest.

57. A given weight P draws another given weight W up an inclined plane of given height and length, by means of a string parallel to the plane; when and where must P cease to act that W may just reach the top?

58. Divide the length of a given inclined plane into three parts, so that the times of descent down them successively may be equal.

59. Determine that point in the hypothenuse of a right-angled triangle whose base is parallel to the horizon, from which the time of a body's descent to the right angle may be the least possible.

60. A body falls down a given inclined plane, and at the instant when it begins to fall, another is projected upwards from the bottom of the plane with a velocity equal to that acquired in falling down a similar plane n times its length. Where will they meet?

61. A body of given elasticity ϵ is projected up an inclined plane with a given velocity, and at the top impinges perpendicularly upon another plane, and returns to the point from which it set out with two-thirds of the velocity of projection. Find the length of the plane, and show that ϵ is less than $\frac{2}{3}$.

62. If from any point in a rectangular hyperbola whose axis is vertical, two lines be drawn to the extremities of this axis; the times of descent down them will be equal.

63. In a hyperbola whose major axis is horizontal, determine the diameters down which a heavy body will descend in a given time; also that diameter down which it will descend in the shortest time.

64. In an inverted parabola the time of descending down any chord from a point P to the lowest point is equal to the time of descending vertically to a horizontal line which is at a distance below the vertex equal to the latus rectum.

65. Two bodies fall from two given points in the same vertical, down two straight lines drawn to any point of a curve in the same time; all the lines are in the same vertical plane; find the equation to the curve.

66. Determine that diameter of a circle down the last half of which a body descends in the same time as down the whole vertical diameter.

67. OA and OB are vertical and horizontal radii of a circle re-

spectively; it is required to find a point C, in the quadrant AB, to which if a tangent be drawn meeting the radius OB produced in D and a line touching the circle at A in the point E; so that the time down DE plus the time of moving along EA with the acquired velocity may be equal to time down the vertical diameter.

68. Two bodies, A and B, descend from the same extremity of the vertical diameter of a circle, one down the diameter, and the other down the chord of 30°. Find the ratio of A to B when their centre of gravity moves along the chord of 120°.

69. Two circles are situated in the same vertical plane; determine analytically and geometrically the straight line of quickest descent from one to the other; and show that the two results agree.

70. The plane of a circle is inclined at any given angle to the horizon; show that the times of descent down any chord from the highest point are the same.

71. A body, whose elasticity $e = \frac{1}{2}$, projected from the floor of a room 12 feet high strikes the ceiling and floor and just reaches the ceiling again; find the velocity of projection.

72. A rocket ascending vertically with a velocity of 100 feet in 1″ explodes when it has reached its greatest height, and the interval between the sound of the explosion reaching the place of starting, and a place $\frac{1}{4}$ mile distant, is 1″. Determine the velocity of sound.

73. A right-angled triangle being placed with its two sides horizontal and vertical respectively, it is required to determine their proportion so that the time of falling down the perpendicular and describing the base with the velocity acquired may be equal to the time of descent down the hypothenuse.

74. Find the straight line of quickest descent from a given point within a circle to the circumference.

75. Find the straight line of quickest descent from a given circle to a given circle within it.

76. Find the straight line of quickest descent from a given circle to a given point, (1) within the circle, (2) without it.

77. Find the straight line of quickest descent from the focus to the curve of a parabola whose axis is vertical.

78. A ball of given elasticity is projected vertically upwards with a velocity of 40 feet in 1″; it returns to the point of projection, which is on a hard horizontal plane, and rebounds; it returns again and rebounds, and so on, till the motion ceases; required the whole space described.

79. Two balls, whose weights are 9 lbs. and 2 lbs. respectively, are connected by a string 15 feet in length; the former weight rests on a smooth horizontal table along which it is drawn by the latter that begins to fall vertically; find the velocity required after falling

through (1) 12 feet, and (2) 20 feet, and the times of motion in both cases.

80. A weight P, after falling freely through a feet, begins to raise up a weight Q connected with it by means of a string passing over a fixed pulley; required the subsequent motion.

81. A mass of 18 lbs. is so distributed at the extremities of a cord passing over a fixed pulley that the more loaded end descends through 13 feet in as many minutes; required the weights at each end.

82. Two equal weights W are suspended over a fixed pulley; what weight must be added to one of them that it may descend through 100 feet in 8″?

83. A weight of 7 lbs. draws up one of 5 lbs. over a fixed pulley; at the instant of letting go the weight of 7 lbs. a velocity 4 feet in 1″ is communicated to it; how far will it descend in 8″, and what velocity will it have acquired at the end of that time?

84. The accelerating force on the centre of gravity of two bodies, P and Q, moving vertically and connected by a string passing over a fixed pulley $= \left(\frac{P-Q}{P+Q}\right)^2 . g$.

85. The major axis of an ellipse is vertical; determine the radius vector measured from the upper focus, down which the time of descent is the least possible.

ON MOTION UPON A CURVE AND ON THE SIMPLE PENDULUM.

General Formula. If l denote the number of inches in the length of a simple pendulum, t'' the time of one oscillation, g the measure of the force of gravity ($=32{\cdot}2$ feet generally), then $t=\pi\sqrt{\frac{l}{g}}$.

The length of a second's pendulum in London $=39{\cdot}1393$ inches.

86. Three planes A, B, C are in contact; A is vertical, B and C are inclined to the horizon at angles 60° and 30° respectively; find the velocity with which a body beginning to descend from A will begin to move along the horizontal plane passing through the lower extremity of the plane C.

87. If an inelastic body move uniformly along one side of a regular polygon, show that it will continue to describe the other sides uniformly, but with velocities decreasing in geometrical progression; and in the case of a hexagon, show that the time of describing the first side : time of describing the last :: 1 : 32.

88. Having given the length of a pendulum that will oscillate

seconds; find the length of a pendulum that will oscillate 4 times in 1''; and another 9 times in 1'.

89. Find the length of a pendulum which oscillates as often in 1' as there are inches in its length.

90. Two pendulums, the lengths of which are L and l, begin to oscillate together, and are again coincident after n oscillations of L. Given L to find l.

91. Find the time of an oscillation of a pendulum 11 feet in length 3 miles above the earth's surface at the equator, where the second's pendulum equals 38·997 inches.

92. A second's pendulum is carried to the top of a mountain and there loses $48''\cdot6$ in a day; determine the height of the mountain, supposing the earth's radius to be 4000 miles.

93. A pendulum which should beat seconds is found to lose 10'' a day. Determine the quantity by which its length should be increased or diminished.

94. The force of gravity varying as $\frac{1}{(\text{dist.})^2}$ from the centre of the earth, how high must a second's pendulum be carried above the level of the sea that it may vibrate $59\frac{1}{2}$ times in 1', the radius of the earth being 3958 miles?

95. The times of oscillation of a pendulum are observed at the earth's surface and at a given depth below the surface; find from these data the radius of the earth which is supposed to be spherical.

96. The length of a pendulum that vibrates sidereal seconds being 38·926 inches; find the length of a sidereal day. Find the increment of the length of the pendulum that it may measure mean solar time.

97. The length of a second's pendulum being 39·06 inches at the equator, 39·28 at the pole, and 39·2 in latitude 52°; find the force of gravity at the equator and at the pole.

98. A second's pendulum is lengthened 1·05 inches; find the number of seconds it will lose in 12 hours.

99. Prove that the times of vibration of the same pendulum when carried to different heights above the earth's surface are to each other as the distances of those heights from the earth's centre.

100. A pendulum gains $0''\cdot05$ in an hour before it is carried up a high mountain; at what height in the ascent will the pendulum keep true time?

ON THE MOTION OF PROJECTILES IN A NON-RESISTING MEDIUM.

General formulæ. If two straight lines be drawn through the point of projection, one horizontal, the other vertical; and these lines be taken for the coordinate axes of x and y respectively; V the velocity of projection, and α the angle which the direction of projection makes with the axis of x; h the impetus which equals the space due to the velocity, and hence $h=\frac{V^2}{2g}$, where g measures the force of gravity; then the equation to the curve described by the projectile is

$$1.\ y=x\tan\alpha-\frac{gx^2}{2V^2\cos^2\alpha}\text{ or }x\tan\alpha-\frac{x^2}{4h\cos^2\alpha}.$$

From this equation most of the properties respecting projectiles may be deduced.

If R be the horizontal range and T″ the time of describing this range, H the greatest height, then

$$2.\ T=\frac{2V}{g}\sin\alpha,\qquad 3.\ R=2h\sin 2\alpha.\qquad 4.\ H=h\sin^2\alpha.$$

If w be the weight of a ball or shell, p the weight of the gunpowder used in firing the ball or shell from a mortar, and v the velocity generated by the powder, then

$$5.\ v=1600\sqrt{\frac{3p}{w}}\text{ feet.}$$

101. If a body be projected with a velocity of 850 feet in 1″ in a direction making an angle of 60° with the horizon; find the focus of the parabola described, and also its latus rectum.

102. A body is projected in a direction making an angle of 15° with the horizon, with a velocity of 60 feet in 1″; find its range, greatest altitude, and time of flight.

103. A body is projected at an angle of 45°, and descends to the horizon at a distance of 500 feet from the point of projection; with what velocity was it projected, what was its greatest altitude, and the whole time of flight?

104. Find the velocity and direction of projection of a ball that it may be 100 feet above the ground at one mile distance, and may strike the ground at two and a half miles.

105. Compare the space described by a projectile in the direction of projection with its vertical fall in the same time. If the velocity be given, determine the angle of projection that the focus may lie in the horizontal line through the point of projection.

106. The horizontal range of a projectile is 1000 feet and the time of flight is 15″; find the direction and velocity of projection; also the greatest altitude of the body during the flight.

107. If the horizontal range of a body projected with a given velocity be three times the greatest altitude, find the angle of projection. What is the value of this angle when the range is equal to the altitude?

108. A shot is fired with a given velocity towards a tower whose

horizontal distance from the cannon is one-half the range, and whose altitude subtends an angle $=\tan^{-1}\frac{3}{4}$ at the point of projection; find the inclination of cannon to the horizon that the shot may strike the summit of the tower.

109. From one extremity of the base (500 feet) of an isosceles triangle, whose vertical angle is 36°, situate in a vertical plane, a body is projected in the direction of the side adjacent to that extremity so as to strike a body placed in the other extremity; find the velocity of projection, and the time of flight.

110. A shell being discharged at an angle of 45°, the sound of its explosion was heard at the mortar $3''\cdot 5$ after the discharge; required the horizontal range, the velocity of sound being $35\,g$ in $1''$.

111. A body is projected at an angle of 60° elevation with a velocity of 150 feet in $1''$; find the direction and velocity of the projectile after the lapse of $5''$; and its height above the horizontal plane passing through the point of projection.

112. Two bodies are projected from the same point with the same velocity; the directions of projection are measured by the angles α and 2α respectively; compare the areas of the parabolas described, supposing the horizontal ranges equal.

113. If the areas in the last problem be equal; what is the value of α?

114. A body is projected with a given velocity from a given point and in a given direction; find where it will strike a given plane.

115. A body is projected from a given point with a given velocity; to find the direction that it may just touch a given plane.

116. A body is projected from the summit of a hill, whose form is a right cone the vertical angle of which is 120°, in a given direction with a given velocity; to find where the projectile will strike the hill.

117. A body is projected from the top of a tower 200 feet high with a velocity of 50 feet in $1''$ and at an angle of elevation $=60°$; find the range on the horizontal plane passing through the foot of the tower, and the time of flight.

118. From the top of a tower two bodies are projected with the same velocity at different given angles of elevation, and they strike the horizon at the same place; to find the height of the tower.

119. Show that a body projected in an oblique direction along an inclined plane describes a parabola, and find its latus rectum, having given the inclination of the plane, also the velocity and direction of projection.

120. A body projected in a direction making an angle of 30° with a plane whose inclination to the horizon is 45°, fell upon the plane at the distance of 250 feet from the point of projection, which

is also in the inclined plane; required the velocity of projection, and the time of flight.

121. At the foot of a tower 60 feet high runs a river 300 feet in width: a hill slopes from the opposite bank of the river at an angle of 30° to the horizon; from the top of the tower a cannon ball is fired at an elevation of 60°; the velocity of projection is equal to that acquired by a body falling from rest through 500 feet: at what distance from the bank of the river will the ball strike the hill?

122. The heights of the ridge and eaves of a house are 40 feet and 32 feet respectively, and the roof is inclined at 30° to the horizon. Find where a sphere falling down the roof from the ridge will strike the ground, and also the time of descent from the eaves.

123. If a body be projected up a plane AC inclined at 45° to the horizon with the velocity acquired in falling down a vertical line $=\text{AC}$; find the range AD on the horizontal plane passing through the point A. Determine also the time between its leaving the point of projection and meeting the horizontal plane.

124. Two bodies A and B are projected at the same instant from the same point with velocities u and v respectively—the one vertically and the other at an elevation of 30°; find the path described by their centre of gravity.

125. Four balls whose weights are 2, 3, 5 and 6 pounds respectively are projected from the same point at the same instant and with the same velocity of 1000 feet in $1''$; the angles of elevation are severally $22° \, 30'$, 30°, 45° and 60°; find the height of their common centre of gravity after $3''$ have elapsed, and the highest point to which it will rise.

126. Several bodies being projected in different directions from the same point and with the same velocity; it is required to find the locus of all the bodies at the end of a given time.

127. Find the locus of the vertices of all the parabolas described under the circumstances of the last problem.

128. Planes AP, AP′, AP″, &c. being drawn in every direction from the point A, and bodies projected from A with a given velocity at such angles that the ranges on each of these planes shall be the greatest; to find the locus of all the extreme points P, P′, P″, &c.

129. A body of given elasticity slides down an inclined plane of given length whose inclination $=\cos^{-1}\frac{1}{\sqrt{3}}$ and impinges on the horizontal plane at the foot of the given plane; required the range of the body after reflection at the horizontal plane.

130. An imperfectly elastic ball from a given height is let fall on a given inclined plane; required the point at which it will again strike the plane after reflection.

131. Two balls are projected at the same instant from two given

points in a horizontal plane and in opposite directions so as to describe the same parabola. What must be their relative magnitude and elasticity so that after impact one of them may return through the same path as before and the other descend in a right line?

132. A perfectly elastic body is projected from a point in a plane whose inclination to the horizon is ι; find the angle of projection in order that after striking the plane the body may be reflected vertically upwards.

133. A circle has chords drawn from the extremity of a diameter which is at right angles to a horizontal plane. Show that, if a body whose elasticity is ·25 descend down the chord whose inclination to the vertical diameter is 30°, and be reflected by the horizontal plane, the range will be the greatest possible; and find this range.

134. AB is the vertical diameter of a circle. A perfectly elastic ball descends down the chord AC, and being reflected by the plane BC, describes its path as a projectile; show that the body will strike the circle at the opposite extremity of the diameter CD.

135. A ball of given elasticity is projected with a given velocity at a given elevation. On meeting the horizontal plane it rebounds, then describes another parabola and again rebounds; and so on. Find the whole horizontal distance described before the ball ceases to rebound.

---

136. A ball of 13 lbs. weight is fired from a mortar on the summit of a mountain with a charge of 6·5 oz. of powder so as just to strike at the bottom and with double the velocity of projection. Find the height of the mountain and the greatest height attained by the projectile.

137. How much powder is required to throw an 8-inch shell 1500 yards on an inclined plane passing through the point of projection, its inclination being 28° 45′, and the inclination of the mortar being 48° 30′?*

138. A gun is mounted on a citadel 450 feet above the level of the sea. A pirate is observed in a line making an angle of depression $= 5^\circ\ 30'$; and it is required to fire into her a 13-inch shell: how much powder will be necessary, and what is the time of flight, the inclination of the gun being 24° 30′?*

---

* The 8-inch and 13-inch shells are supposed to weigh 48 lbs. and 196. lbs respectively.

ANSWERS.

ARITHMETIC.

VULGAR FRACTIONS.

1. $\frac{5}{12}$, $\frac{8}{27}$, $\frac{7}{90}$.
2. $\frac{7}{8}$, $\frac{13}{15}$, $\frac{3}{17}$, $\frac{2}{7}$.
3. $\frac{10}{11}$, $\frac{9}{11}$, $\frac{355}{113}$, $\frac{99991}{97073}$.
4. $\frac{16807}{59049}$, and $\frac{21}{29}$.
5. $\frac{13}{12}$, $\frac{500}{9}$, $\frac{14443}{8}$, $\frac{2519}{120}$.
6. $\frac{58}{15}$, $\frac{96}{17}$, $\frac{167}{19}$, $\frac{179}{18}$.
7. $\frac{293}{21}$, $\frac{532}{19}$, $\frac{120290}{1191}$, and $\frac{62312523}{3817}$.
8. $1\frac{1}{4}$.
9. 1.
10. $\frac{1}{7}$.
11. $41\frac{11}{20}$.
12. $4771\frac{1}{12}$.
13. $\frac{5}{18}$.
14. $1\frac{13}{105}$.
15. $2\frac{43}{60}$.
16. $\frac{9}{560}$.
17. $2\frac{4}{5}$.
18. $\frac{1}{8}$.
19. $4\frac{5}{7}$.
20. $\frac{465}{121808}$.
21. 3.
22. $\frac{33}{140}$.
23. 9.
24. $5\frac{1}{3}$.
25. $2\frac{1}{5}$.
26. $3\frac{2}{3}$.
27. $\frac{10}{21}$, 7, $11\frac{77}{108}$, and 2.
28. $8\frac{10}{11}$.
29. $\frac{11}{135}$.

30. $\frac{11}{112}$.
31. $\frac{113}{123}$.
32. $\frac{64}{81}$.
33. $\frac{53}{122}$, and $\frac{7}{11}$.
34. 1, and $15\frac{1573}{7380}$:
35. $\frac{2}{3}$, and $2\frac{13}{16}$.
36. $\frac{5}{57}$.
37. $\frac{1}{3}$.
38. 1.
39. $4\frac{109549}{410000}$.
40. $2\frac{11}{14}$.
41. $5\frac{94}{399}$.
42. $6\frac{97}{132}$.
43. $\frac{68}{157}$, and $\frac{82}{151}$.
44. $\frac{1}{336}$.
45. 2*s.* 3*d.*
46. £2 17*s.* $7\frac{1}{5}$*d.*
47. £2 10*s.* 9*d.*
48. 7*s.* $7\frac{3}{4}$*d.*
49. £2 16*s.* 3*d.*
50. $\frac{3}{4}$th of a farthing.
51. $\frac{19}{50}$.
52. $\frac{1}{7}$.
53. $1\frac{3}{5}$.
54. $\frac{7}{18}$.
55. $\frac{4}{5}$.
56. $\frac{379}{480}$.
57. $\frac{1}{4}$.
58. $\frac{2639}{8748}$.
59. £28 14*s.* $8\frac{9}{20}$*d.*
60. $\frac{12}{95}$th of a farthing.
61. 2*d.*
62. 17*s.* 6*d.*; 16*s.* 4*d.*; and $\frac{7}{30}$.
63. 13 dwts. $13\frac{35}{121}$ grs.
64. 7 oz. 4 dwts.
65. 7 cwt. 3 qrs. 14 lbs.
66. $\frac{7}{9}$.
67. $\frac{11}{36}$.
68. $\frac{17}{5280}$.
69. 2 roods 20 poles.
70. 12 h. 51 m. $25\frac{5}{7}$ sec.
71. 9 h. 10 m. $17\frac{1}{7}$ sec.
72. $\frac{4621}{17532}$.

DECIMAL FRACTIONS.

73. 31·0187.
74. $39\cdot924\dot{9}\dot{8}$.
75. 1561·05.
76. 17·09582248.
77. ·000376, and ·0000037128.
78. ·43204577, and 21·3978.
79. 267, ·0104, and 750.
80. ·00694, $\cdot01\dot{6}$, and 5000.
81. 1240, 220, and 1989·2092.
82. 30000, ·001, and 1240.
83. ·08, ·00125, and ·0078125.
84. ·625, ·008125, ·5136, and ·0448.
85. 2·1875, and ·98.
86. $\cdot\dot{3}$, $\cdot\dot{1}4285\dot{7}$, $\cdot0\dot{6}$, and $\cdot18\dot{7}$.
87. $\dot{1}\dot{3}$, $2\cdot\dot{3}4\dot{5}$, and $\cdot12\dot{3}4\dot{5}$.
88. $\frac{1}{4}$, $\frac{29}{40}$, and $\frac{817}{5000}$.
89. $1\frac{9}{25}$, $3\frac{291}{500}$, and $15\frac{13}{50}$.
90. $\frac{17}{200}$, $\frac{7}{800}$, and $\frac{4707}{20000000}$.
91. $\frac{2}{9}$, $\frac{3}{11}$, and $\frac{8}{33}$.
92. $\frac{4669}{900}$, $\frac{17}{330}$, and $\frac{241}{27500}$.
93. $3\cdot\dot{6}$, 75, and $1\cdot1\dot{4}\dot{5}$.
94. $\cdot0\dot{5}\dot{2}$, 49, and $301\cdot\dot{7}1428\dot{5}$.
95. 2·7182818.
96. 3·14159265 35.
97. ·0953125.
98. ·405465.
99. 3·141592.
100. ·693147.
101. ·4655.
102. ·91875.
103. $\cdot\dot{5}385024691\dot{3}$.
104. ·00625.
105. $\cdot232\dot{5}\dot{7}$.
106. $\cdot148958\dot{3}$.
107. $\cdot4097\dot{2}$.
108. $\cdot0458\dot{3}$.
109. ·0390625.
110. ·04307015.
111. $\cdot\dot{7}$.
112. ·25.
113. ·2.
114. ·90044932.
115. ·07015.
116. ·42.
117. ·42.
118. 17*s.* 6¾*d.*
119. 19*s.* 7·57*d.*
120. 7*s.* 10½*d.*
121. 9·0522*d.*
122. £1 15*s.* 11·088*d.*
123. 19*s.* 5⅓*d.* nearly.
124. 3*s.* 6*d.* or £·175.
125. 13·9968 grains.
126. 10·1376 drams.
127. 1 ft. $5\frac{1}{10}$ in.
128. 14 po. 2 yds. 2 ft. 5·376 in.
129. 6 furlongs, 4 poles, 4 yds. 1 ft. $2\frac{2}{5}$ in.
130. 8 perches.
131. 7½*d.*
132. 1*s.* 3¾*d.*
133. £1 2*s.* 9¾*d.*
134. 5*s.* 7·012*d.*
135. 2*s.* 11¼*d.*
136. 3 cwts. 2 qrs. 26 lbs. $\frac{4}{5}$ oz.
137. 2·30075 feet.
138. 3 furlongs, 25 poles.
139. 3 furlongs, 10 poles, 3 yds. 2 ft.

EXTRACTION OF ROOTS.

140. 51, 85, and 99.
141. 625, 2401, and 6561.
142. 1·4142, and 87·5099.
143. 8·6802, ·6576, and 29·606.
144. ·027, and 6·0000005208.
145. 2·3094, $1{\cdot}\dot{3}$, $2{\cdot}4\dot{9}$, and 42·291.
146. 46, and 74.
147. 98, and 512.
148. ·9226, and 1·03.
149. ·192, and ·0386.
150. 9, and 36.
151. 9·05538, and 1·0583005.
152. 9, and 8.
153. 5·1961524.
154. $\frac{5}{6}$, and $5\frac{1}{4}$.
155. $5\frac{2}{5}$, and $16\frac{7}{12}$.
156. ·881917, ·69189, and ·1175019.
157. ·24253, ·31943, and 3·952847.
158. 2·9032, 8·764, and 20·4939.
159. 1·2966, and 1·81.
160. 8·2207, and ·9859.
161. ·57735, and ·267949.
162. 3·968118, and ·288982.
163. 6·707106, and 2·245365.
164. 1·207106, and 6·162277.
165. 11·916079, and ·001097.
166. 1·837117, and 1·551.
167. 3·2305, and 1·241629.
168. −·033124, and 7·231.
169. $5{\cdot}041\dot{6}$, and 28·940474.
170. ·614298, and ·003851.

DUODECIMALS.

171. 42 ft. 1′ 8″.
172. 436 ft. 11′.
173. 92 ft. 2′ 5″.
174. 1627 ft. 4′ 6″.
175. 28 ft. 5′ 6″ 8‴.
176. 107 ft. 4′ 6″ 9‴ 8^{iv}.
177. 3252 ft. 7′ 2″ 1‴.
178. 80 ft. 11′ 8″.
179. 1375 ft. 2′ 11″ 3‴.

DIRECT SIMPLE PROPORTION.

180. £212 10*s.*
181. 500 weeks.
182. £9 15*s.*
183. 60 dozen.
184. £20 8*s.* 4*d.*
185. 13*s.* $4\frac{896}{5773}$*d.*
186. 2 qrs. 10 oz. $4\frac{4}{7}$ dr.
187. He lost 4 pence.
188. I neither gain nor lose.
189. 5 shillings.
190. I lost 7 shillings.
191. 301 yards.
192. 11 guineas.
193. £11 5*s.* $4\frac{136}{191}$*d.*
194. £953 10*s.* $3\frac{723}{839}$*d.*
195. $6\frac{10860}{16493}$ pence.
196. $67\frac{8}{11}$ gallons.
197. 104 guineas.
198. $13\frac{1}{3}$ years.
199. £3 17*s.* $10\frac{1}{2}$*d.*

200. $9\frac{1}{11}$ dwts.
201. $33\frac{41}{55}$ minutes.
202. $5\frac{11}{21}$ gallons.
203. £36 6*s.* $\frac{1}{24}$*d.*
204. $82\frac{2}{3}$ per cent.
205. £90.
206. $10\frac{5}{7}$ per cent.
207. $7\frac{1}{2}$ per cent. and 3 dwts. $8\frac{19}{40}$ grs.
208. 1457 francs $62\frac{1}{2}$ cent.
209. £24 11*s.* 10·35*d.*
210. $7908\frac{102}{121}$ miles.
211. $111835{\cdot}\dot{4}2857\dot{1}$ metres.
212. $109363923\frac{3}{4}$ yards.
213. 206264·808 seconds.
214. 20853231·98 feet.
215. 32·19 inches.
216. 3·104 days.

INVERSE SIMPLE PROPORTION.

217. $6\frac{6}{7}$ months.
218. 18 feet.
219. $12\frac{12}{19}$ days.
220. $97\frac{7}{9}$ yards.
221. $4166\frac{2}{3}$ yards.
222. $63\frac{13}{18}$ yards.
223. $214\frac{2}{7}$ miles.
224. 7·1 days.
225. $42\frac{14}{33}$ paces.
226. 260 paces.
227. $8\frac{19}{22}$ months.

COMPOUND PROPORTION.

228. 9 days.
229. 18 days.
230. £613 6*s.* 8*d.*
231. £453 12*s.*
232. 10 days.
233. £11 0*s.* $2\frac{2}{39}$*d.*
234. £15 16*s.* $\frac{26}{27}$*d.*
235. £2000.
236. $23\frac{41}{408}$ loads.
237. 200 of A.
238. Mohur of Bengal, £1 13*s.* 6·82*d.*
Mohur of Bombay, £1 10*s.* 1·29*d.*
Bombay gold rupee, £1 9*s.* 2·52*d.*
Madras gold rupee, £1 9*s.* 2·18*d.*
Star pagoda, 7*s.* 4·31*d.*
239. Sicca rupee, 2*s.* 0·607*d.*
Arcott rupee, 1*s.* 11·258*d.*
Bombay rupee, 1*s.* 11·146*d.*
Baroch rupee, 1*s.* 9·948*d.*
240. 132 days.

PRACTICE.

241. £2 15*s.* $3\frac{1}{2}$*d.*
242. £12 3*s.* $5\frac{1}{4}$*d.*
243. £55 15*s.* 11*d.*
244. £1925 6*s.* $6\frac{1}{4}$*d.*
245. £93 19*s.* 11*d.*
246. £1906 8*s.* 9*d.*
247. £529 11*s.* $\frac{1}{2}$*d.*
248. £1603 0*s.* $3\frac{3}{4}$*d.*
249. £79 4*s.*
250. £937 16*s.*

251. £17478 11*s.*
252. £41101 5*s.*
253. £469 14*s.*
254. £4897 1*s.* 3*d.*
255. £696 1*s.* 11*d.*
256. £16718 3*s.* 10$\frac{1}{4}$*d.*
257. £993 15*s.*
258. £2540 6*s.*
259. £598 5*s.* 7$\frac{1}{2}$*d.*
260. £2770 9*s.* 11*d.*

SIMPLE INTEREST.

261. £13 12*s.* 7$\frac{11}{25}$*d.*, and £354 8*s.* 1$\frac{11}{25}$*d.*
262. £35 7*s.* 1$\frac{4}{5}$*d.*, and £271 1*s.* 5$\frac{4}{5}$*d.*
263. £230 1*s.* 5$\frac{33}{100}$*d.*, and £1873 8*s.* 10$\frac{83}{100}$*d.*
264. £668 14*s.* 5·131*d.*, and £3529 11*s.* 2·881*d.*
265. £10 8*s.* 6*d.*, and £357 18*s.* 6*d.*
266. £91 7*s.* 10$\frac{6}{11}$*d.*
267. £483 17*s.* 5$\frac{1}{31}$*d.*
268. 180 years.
269. 28$\frac{4}{7}$ years.
270. 47$\frac{1}{17}$ years.
271. 2$\frac{8}{21}$ per cent.
272. 3$\frac{1}{3}$ per cent.

COMPOUND INTEREST.

273. £55 5*s.* 6*d.*
274. £202 15*s.* 1$\frac{1}{2}$*d.*
275. £682 15*s.* 7*d.*
276. £23 15*s.* 2*d.*
277. £77 11*s.* 11*d.*
278. £980 12*s.* 4$\frac{1}{2}$*d.*
279. (1) £884 9*s.* 2$\frac{1}{2}$*d.*
(2) £887 12*s.* 7$\frac{1}{4}$*d.*
(3) £889 4*s.*
280. £473 19*s.* 4$\frac{3}{4}$*d.*
281. £3223 6*s.*
282. £123096, &c. to 37 places of integers.
283. £613 18*s.* 4*d.*
284. ·014613 farthings.
285. £13 12*s.* 4$\frac{3}{4}$*d.* per cent.
286. £12 4*s.* 11*d.* per cent.
287. 47·303 years.
288. 36·894 years.

DISCOUNT.

289. £166 13*s.* 4*d.*
290. £21 16*s.* 10$\frac{1}{2}$*d.*
291. £21 3*s.* 4$\frac{1}{3}$*d.*
292. £296 6*s.* 2$\frac{1}{2}$*d.*
293. £425 10*s.* 7$\frac{3}{4}$*d.*
294. £17 8*s.* 3$\frac{1}{4}$*d.*
295. £806 17*s.* 1*d.*

PURCHASE OF STOCKS.

296. £9375 purchase, and £3 4*s.* per cent.
297. £531 18*s.* 3$\frac{1}{2}$*d.* stock, and £18 12*s.* 4*d.* income.
298. £3 8*s.* 11$\frac{3}{4}$*d.* per cent.
299. £53120 16*s.* 11$\frac{3}{4}$*d.* stock.
300. £7600 purchase, and £6 11*s.* 6$\frac{3}{4}$*d.* per cent.
301. £6 12*s.* 2$\frac{3}{4}$*d.* per cent.

302. £800 stock.

303. £1444 10*s*. $2\frac{7}{10}$*d*. purchase.

304. £821 5*s*. purchase.

305. £67 15*s*. $11\frac{1}{5}$*d*. income.

306. $89\frac{63}{80}$.

307. £408 6*s*. $0\frac{1}{2}$*d*. purchase.

SINGLE FELLOWSHIP.

308. A's share £10 5*s*. $10\frac{10}{17}$*d*.
B's share £11 15*s*. $3\frac{9}{17}$*d*.
C's share £13 4*s*. $8\frac{8}{17}$*d*.
D's share £14 14*s*. $1\frac{7}{17}$*d*.

309. £136 11*s*. $11\frac{49}{97}$*d*.
£163 18*s*. $4\frac{20}{97}$*d*.
£229 9*s*. $8\frac{28}{97}$*d*.

310. A £666 13*s*. 4*d*.
B £333 6*s*. 8*d*.
C £166 13*s*. 4*d*.
D £83 6*s*. 8*d*.

311. $41\frac{2}{3}$, $31\frac{11}{24}$, and $26\frac{7}{8}$.

312. 32, 33, 34, 35, 36, and 37 men.

DOUBLE FELLOWSHIP.

313. A £212 2*s*. $6\frac{210}{409}$*d*.
B £194 8*s*. $11\frac{397}{409}$*d*.
C £176 15*s*. $5\frac{175}{409}$*d*.
D £139 13*s*. $0\frac{36}{409}$*d*.

314. The captain £251 10*s*. $6\frac{1914}{4421}$*d*.
Each lieutenant £98 16*s*. $0\frac{1248}{4421}$*d*.
Each sergeant £76 0*s*. $0\frac{960}{4421}$*d*.
Each corporal £26 1*s*. $1\frac{3487}{4421}$*d*.
Each private £8 15*s*. $8\frac{2436}{4421}$*d*.

ALGEBRA.

1. 0.

3. 22.

4. $(a+b)x^2-(b-7)y^2$.

5. $x+4$.

6. ax.

7. $a-4+4x$.

8. $(a-b)\{x^3-2ax^2+a^2x-(a+b)b^2\}$.

9. $a^4+a^2x^2+x^4$.

10. $a^2-b^2-c^2-d^2-2bc-2bd+2cd$.

11. $x^5-(a+t)x^4+(b+at+s)x^3-(c+bt+as)x^2+(ct+bs)x-cs$.

12. $a^3+b^3+c^3-3abc$.

13. $5x^4+\frac{7}{2}ax^3-\frac{107}{12}a^2x^2-\frac{5}{6}a^3x+\frac{7}{6}a^4$.

14. $xy(-x^2+y^2+z^2-yz)+xz(x^2-z^2)+2yz(y^2-z^2)$.

15. $$\left\{\begin{array}{l}-3a^3cd+a^2b^2d+4a^2bc^2-3a^2bcd-3a^2c^2d-3a^2cd^2-2abcd^2\\ \quad+ab^3d+ab^2cd+4ab^2c^2+4abc^3+ab^2d^2+4abc^2d-2b^2cd^2\\ \quad-2bc^2d^2-2bcd^3.\end{array}\right.$$

16. $2a^{2m}+2a^mb^p-4a^mc^n-3a^mb-3b^{p+1}-6bc^n$.

17. $x^4+2x^3-41x^2-42x+360$. 18. $\frac{281}{1260}$.

19. x^3. 20. $x^{2^n}-1$.

DIVISION.

21. $-\frac{3a^2bf}{2e}-\frac{2b^2de}{cg}+\frac{b^3d^3f^2}{2aceg}-\frac{2c^2deg^3}{3ab}$.
22. $x^4+2ax^3+3a^2x^2+2a^3x+a^4$.
23. $3h^3-5h^2k+2hk^2$. 24. $a^4+3a^3+9a^2+27a+81$.
25. $p+2q-r$. 26. $x^3+12x^2-43x+30$.
27. $1+5x+15x^2+45x^3+135x^4+\ldots$
28. $x^2-2xy+\frac{y^2}{2}$. 29. $(x^2+x+1)a-(x+1)$.
30. bx^2+cx-f. 31. $\frac{5}{6}a+\frac{3}{4}b+5c$.
32. $2x^3y^{-3}-3x^4y$.
33. $x^{pq-p}+x^{pq-2p}+\ldots+x^{2p}+x^p+1$.
34. $(b-c)a^{n-2}+(b^2-c^2)a^{n-3}+(b^3-c^3)a^{n-4}+\ldots+(b^{n-2}-c^{n-2})a$
$+(b^{n-1}-c^{n-1})$.

GREATEST COMMON MEASURE.

35. $a+3b$. 36. $3x-2$.
37. x^2-3. 38. $x^3-3x^2y+3xy^2-y^3$.
39. $x+a$. 40. $2a^2x-3ay^2$.
41. $2a+3b-c$. 42. e^x+1.
43. $3x-y$.

LEAST COMMON MULTIPLE.

44. $72a^2x^5$. 45. $a^4-a^3x+ax^3-x^4$.
46. $x^5-3x^4-19x^3+3x^2+18x$.
47. $21x^4-26x^3-55x^2+78x-24$.
48. $2x^4+(2a-3b+2c)x^3-(3ab-2ac+2b+3bc)x^2$
$-(3abc-3b^2+2bc)x+3b^2c$.
49. $x^{10}-x^6-x^4+1$.

FRACTIONS.

50. $\frac{x}{4x^2-y^2}$. 51. 1.

52. a. 53. $\frac{a+bx}{b+ax}$.

54. $\dfrac{85x-20y}{84}$.

55. $\dfrac{1}{1-x}$.

56. $\dfrac{1}{1-x^4}$.

57. $\dfrac{1+x+x^2}{1-x-x^4+x^5}$.

58. $\dfrac{x+c}{(x-a)(x-b)}$.

59. $\dfrac{1}{(x+1)(x+3)}$.

60. $\dfrac{x}{(x+1)(x+2)(x+3)}$.

61. $\dfrac{1}{(x-a)(x-b)(x-c)}$.

62. $\dfrac{x}{(x-a)(x-b)(x-c)}$.

63. $\dfrac{x^2}{(x-a)(x-b)(x-c)}$.

64. $\dfrac{x^2+hx+k}{(x-a)(x-b)(x-c)}$.

65. $x^{2n}+2$.

66. $\dfrac{23+16x-30x^2-3x^3}{6-11x-21x^2-x^3+3x^4}$.

67. $\dfrac{3-3x-x^2+2x^3-x^4+x^5-x^6}{1+x+x^2+2x^3+x^4+x^5+x^6}$.

68. $\dfrac{3+3x-x^2-6x^3-9x^4-7x^5-3x^6}{1+3x+5x^2+6x^3+5x^4+3x^5+x^6}$.

69. $\dfrac{x^2}{y}$.

70. $\dfrac{bcd}{a}$.

71. $\dfrac{a^4-x^4}{a^2x}$.

72. $\dfrac{a-x}{a+2x}$.

73. $\dfrac{x^2-xy+yz-z^2}{y}$.

74. $\dfrac{ax(a^2-ax-x^2)}{a^2-x^2}$.

75. $\dfrac{a^4+a^2x^2+x^4}{a^4-x^4}$.

76. $\dfrac{3a^4}{x^4}-\dfrac{19a^3b}{10x^3y}+\dfrac{21a^2b^2}{5x^2y^2}-\dfrac{9ab^3}{10xy^3}+\dfrac{b^4}{y^4}$.

77. $\dfrac{a^4}{b^6}-\dfrac{4c^6d^8}{b^{10}}-\dfrac{14c^5d^4}{a^4b^8}-\dfrac{49c^4}{4a^8b^6}$.

78. $\dfrac{25a^2d}{3bc^2}$.

79. $\dfrac{ac}{ab-bx}$.

80. $\dfrac{4x^3-3xy^2}{9x^3+8y^3}$.

81. $\dfrac{fh}{bd}\cdot\dfrac{ad+bc}{eh-fg}$.

82. $\dfrac{twy}{bdf}\cdot\dfrac{adf-bcf+bde}{swy+twx+tyz}$.

83. 1.

84. $\dfrac{x^2+y^2}{2xy}$.

85. $\dfrac{1}{2x^2-1}$.

86. $\dfrac{a}{c}-\dfrac{b}{d}+\dfrac{c}{e}$.

87. $\dfrac{acx^2}{bd}+\dfrac{bx}{cd}+\dfrac{a}{b}$.

88. $\dfrac{x-1}{x+1}$.
89. $\dfrac{7a}{5x}$.
90. $\dfrac{11x}{x-a}$.
91. $\dfrac{c+y}{f+2x}$.
92. $\dfrac{x^2-7x+10}{x-10}$.
93. $\dfrac{c}{2df}\cdot\dfrac{ab+9b-5c}{6a+3b-5c}$.
94. $\dfrac{cx+d}{ax+b}$.
95. $\dfrac{5x^2+1}{x(9x^2-4)}$.
96. $\dfrac{4a}{7x^2}\cdot\dfrac{a+3b}{b+c}$.
97. $\dfrac{x^2-11x+28}{x^2}$.
98. $\dfrac{x+2}{x+3}$.
99. $rs+(rt+qs)x+qtx^2$.
100. $\dfrac{a-bx}{a+bx}$.
101. $\dfrac{2y}{b}\left(\dfrac{x}{a}-\dfrac{z}{c}\right)$.
102. $\dfrac{x^3+5x^2-29x-105}{3x^2+10x-29}$.
103. $\dfrac{x+3x^2+2x^3+x^4}{1+4x+3x^2+2x^3}$.
104. 1.
105. -3.
106. 3.
107. 2.
108. -1.
109. xy.
110. n^3+1.
112. $2(1+a^2+a^4)$.
114. $1-6x+21x^2-44x^3+63x^4-54x^5+27x^6$.
116. $(a+b+c)^3$.
117. $e^{3x}-3e^x+3e^{-x}-e^{-3x}$.
118. $p^5x^5\mp 5p^4qx^4z+10p^3q^2.x^3z^2\mp 10p^2q^3x^2z^3+5pq^4xz^4\mp q^5z^5$.
119. $2x^2-3x+4$.
120. $4b^2-2ax+2x^2$.
121. $3x-5a-\dfrac{a^2}{2}$.
122. $x+\dfrac{a}{3}-\dfrac{b}{2}$.
123. $3x-y+5z+t$.
124. $\dfrac{2x}{7y}-5+\dfrac{3y}{4x}$.
125. $\dfrac{x^2}{2y^2}+1+\dfrac{2y^2}{x^2}$.
126. $\dfrac{x}{y}-\dfrac{1}{2}-\dfrac{y}{x}$.
127. $\dfrac{x}{2}+\dfrac{2}{3x^2}$.
128. $x-4+\dfrac{2}{x}$.
129. $(a+1)^{2n}x-2ca^p$.
130. $p^2s=r^2$, $4q=p^2+8\sqrt{s}$.

SURDS.

131. $6a\sqrt{x}$, or $2a\sqrt{x}$. 132. $3a^2\sqrt{3b}$.

133. $a^2b^{\frac{4}{3}}$. 134. $\sqrt[4]{2a}$ and $\sqrt[5]{7a^2}$.

135. $\sqrt{2}$. 136. $\frac{27}{2}\sqrt{3}$.

137. $-\sqrt[3]{9}$. 138. $11\sqrt{3}$.

139. $5^{\frac{4}{3}}$. 140. $\frac{27ab-4a^2}{18b}\sqrt[3]{b}$.

141. $2\sqrt[6]{a^{13}}$. 142. $\sqrt[9]{\frac{1}{a^{14}}}$.

143. $\frac{a^{\frac{3}{4}}}{x^{\frac{1}{2}}}-\frac{a^{\frac{25}{12}}}{x^{\frac{5}{4}}}+\frac{a^{\frac{13}{12}}}{x^{\frac{1}{4}}}-a^{\frac{1}{12}}x^{\frac{3}{4}}$. 144. $\sqrt[6]{\frac{y}{x}}$.

145. $x^2+6xz^{\frac{1}{3}}-4y+9z^{\frac{2}{3}}$. 146. $a-b$.

147. a^3-b^2. 148. $x^2-3x^{\frac{4}{3}}y^{\frac{2}{3}}+3x^{\frac{2}{3}}y^{\frac{4}{3}}-y^2$.

149. $\sqrt[12]{x^{11}y^{13}}+\sqrt[15]{x^{13}y^{17}}-\sqrt[15]{x^{17}y^{13}}-\sqrt[20]{x^{21}y^{19}}$.

150. $\frac{1}{2}\sqrt[4]{a^3}+\frac{1}{2}\sqrt[12]{a^7}+\frac{1}{4}\sqrt{a}-\frac{3}{10}\sqrt[10]{a^7}-\frac{1}{5}\sqrt[15]{a^8}-\frac{3}{20}\sqrt[20]{a^9}$.

151. $\frac{1-a^{\frac{1}{2}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}}$. 152. $px^3-ax^2-a^2x-(p-2)a^3$.

153. $-\sqrt{ab}$, $\sqrt{-1}.\sqrt[4]{ab}$, $\sqrt[3]{-1}.\sqrt[6]{ab}$.

154. $x^4+2x^3-8x^2-6x-1$. 155. 16.

156. $(a^2-b^2)^{\frac{m+n}{mn}}$. 157. $16-\sqrt{-3}$.

158. x^6+a^6.

159. $-8x^{\frac{3}{2}}yz^{\frac{3}{4}}$, $16x^2y^{\frac{4}{3}}z$, $64x^3y^2z^{\frac{3}{2}}$.

160. $-243x^{10}y^2z^{15}$. 161. $a^{\frac{21}{5}}b^{\frac{7}{5}}c^{\frac{7}{30}}$.

162. $a^{-\frac{11}{5}}$.

163. $2\sqrt{-y^2}-3(x^2+y^2)\{(x+\sqrt{-y^2})^{\frac{1}{3}}-(x-\sqrt{-y^2})^{\frac{1}{3}}\}$.

164. $a+b-c+3\sqrt[3]{a^2b}-3\sqrt[3]{a^2c}+3\sqrt[3]{ab^2}+3\sqrt[3]{ac^2}-3\sqrt[3]{b^2c}$
$+3\sqrt[3]{bc^2}-6\sqrt[3]{abc}$.

165. $\frac{a^{\frac{5}{4}}}{18}$. 166. $x-x^{\frac{1}{2}}$.

167. $a^2+ab^{\frac{2}{3}}+b^{\frac{4}{3}}$. 168. $8x^{\frac{3}{4}}+2x^{\frac{1}{2}}y+\frac{1}{2}x^{\frac{1}{4}}y^2+\frac{1}{8}y^3$.

169. $a^{\frac{1}{4}}-b^{\frac{1}{2}}$.

170. $a^{\frac{5}{3}}+a^{\frac{4}{3}}b^{\frac{1}{6}}+ab^{\frac{1}{3}}+a^{\frac{2}{3}}b^{\frac{1}{2}}+a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{5}{6}}$.

171. $a^{\frac{9}{4}}-a^{\frac{3}{2}}b^{\frac{3}{4}}+a^{\frac{3}{4}}b^{\frac{3}{2}}-b^{\frac{9}{4}}$. 172. $x^{\frac{3}{2}}+2xy^{\frac{1}{2}}+4x^{\frac{1}{2}}y+8y^{\frac{3}{2}}$.

173. $x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}-\frac{1}{2}x^{-\frac{5}{2}}+\frac{1}{4}x^{-\frac{7}{2}}+\ldots$

174. $x^{\frac{1}{4}}+x^{-\frac{1}{4}}y^{\frac{1}{2}}+x^{-\frac{3}{4}}y+x^{-\frac{5}{4}}y^{\frac{3}{2}}+\ldots$

175. $\sqrt[3]{p}.\sqrt[4]{q}-r\sqrt[5]{p}\,\sqrt[3]{q}.$

176. $x^{\frac{n-1}{n}}.+x^{\frac{n-2}{n}}.a^{\frac{1}{n}}+x^{\frac{n-3}{n}}.a^{\frac{2}{n}}+\ldots\ldots+x^{\frac{1}{n}}.a^{\frac{n-2}{n}}+a^{\frac{n-1}{n}}.$

177. $\frac{1}{3}\sqrt[5]{40}.$

178. 1.

179. $x^{\frac{3n-2m}{mn}}.y^{n-m}.$

180. $a^{\frac{1}{4}}+6ab^{\frac{1}{3}}c^{\frac{1}{12}}.$

181. $\frac{2\sqrt{a^2-x^2}}{x^2}.$

182. $\frac{x-1}{\sqrt[3]{x}}.$

183. $\frac{3x^2}{x^3-1}.$

184. $\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}.$

185. $1-\frac{3}{4}x^{\frac{1}{3}}+x^{\frac{1}{2}}.$

186. $\frac{1}{1-x^2}.$

187. $4x\sqrt{x^2-1}.$

188. $\sqrt{x^2+1}+\sqrt{x^2-1}.$

189. $\frac{x^3-2x^2+3x-4}{x^4-x^3+x^2-x+1}.$

190. $\frac{a(b\pm\sqrt{bc})}{b-c}.$

191. $\frac{\sqrt{ab}}{a-c}\left\{\frac{a^2}{\sqrt{3}}+bc\right\}.$

192. $\left(\frac{b^{\frac{1}{n}}}{a^{\frac{1}{m}}}\right)^{pq}.$

193. $\sqrt[12]{a^5b^{17}}.$

194. $\sqrt{3ax^{\frac{1}{2}}}$, and $\sqrt[3]{3ax^{\frac{1}{3}}}.$

195. $(-1)^{\frac{1}{m}}2^{\frac{1}{m^2}}ab^{\frac{2^m}{m}}c^{\frac{2}{m}}.$

196. $a^{\sqrt{2}}-a^{-\sqrt{2}}.$

197. $1-\frac{3}{4}\sqrt{a}+a.$

198. $\frac{3}{2}a^{\frac{3}{2}}-\frac{5}{3}ab^{\frac{1}{2}}+\frac{2}{5}a^{\frac{1}{2}}b.$

199. $\sqrt{x^m}-\frac{1}{2}\sqrt[n]{b}\,\sqrt[p]{x^2}.$

200. $\sqrt[x]{a^{-\frac{1}{y}}}-\sqrt[r]{a^{\frac{1}{y}}b^{\frac{1}{2}}}.$

---

201. $\sqrt{5}+\sqrt{2}.$

202. $\sqrt{\frac{1}{2}}+\sqrt{\frac{3}{2}}.$

203. $2+\sqrt{3}.$

204. $3+\sqrt{7}.$

205. $1+\sqrt{7}.$

206. $5-\sqrt{3}.$

207. $\sqrt{5}-\sqrt{3}.$

208. $6-\sqrt{5}.$

209. $5-2\sqrt{3}.$

210. $\sqrt{\frac{7}{2}}-\sqrt{\frac{1}{2}}.$

211. $2-\sqrt{-3}.$

212. $\sqrt{-1}-\sqrt{-2}.$

213. $1-\sqrt{-3}.$

214. $2+\sqrt{-5}.$

215. $5-2\sqrt{-3}.$

216. $4\sqrt{-1}+3.$

217. $2-\sqrt{-25}.$

218. $\sqrt{-2}+1.$

219. $3-2\sqrt{-1}$.
220. $-1+2\sqrt{-1}$.
221. $1+\sqrt{-1}$.
222. $3-3\sqrt{-1}$.
223. $0{\cdot}2\pm0{\cdot}1\sqrt{-1}$.
224. $x+\sqrt{a^2-x^2}$.
225. $x-\sqrt{xy-x^2}$.
226. $\sqrt{x+y}+\sqrt{z}$.
227. $\sqrt{\frac{1+m}{2}}+\sqrt{\frac{1-m}{2}}$.
228. $1-x+\sqrt{1+2x-x^2}$.
229. $a-\sqrt{ax-x^2}$.
230. $\sqrt{a+b}+\sqrt{a-b}$.
231. $\sqrt{a+b}-\sqrt{c+d}$.
232. $m+n+(m-n)\sqrt{-1}$.
233. $\sqrt{n}+\sqrt{-n}$.
234. $\sqrt[4]{6}(1+\sqrt{2})$.
235. $\sqrt[4]{3}(1+\sqrt{2})$.
236. $\sqrt[4]{2}(1-\sqrt{2})$.
237. $\sqrt[4]{8}\left(\frac{1}{2}+\sqrt{\frac{1}{2}}\right)$.
238. $\sqrt[4]{3}(\sqrt{5}-\sqrt{3})$.
239. $\sqrt[4]{21}\left(1-\frac{1}{7}\sqrt{7}\right)$.
240. $\sqrt{3}-\sqrt{2}-1$.
241. $1+\sqrt{3}+\sqrt{5}$.
242. $1-\sqrt{2}+\sqrt{3}-\sqrt{6}$.
243. 6.
244. $2+\sqrt{5}$.
245. $1+\sqrt{7}$.
246. $\sqrt{21}-1$.
247. $1-\sqrt{2}$.
248. $1-\sqrt{3}$.
249. $\sqrt{2}+\sqrt{3}$.
250. $1+2\sqrt{-1}$.
251. $2-\sqrt{-2}$.
252. $2+\sqrt{3}$.
253. $\sqrt{2}+\sqrt{3}$.
254. $\sqrt{5}+2\sqrt{-1}$.
255. $\sqrt[4]{2}\left(\sqrt{\frac{1}{2}}+\sqrt{\frac{3}{2}}\right)$.
256. $\sqrt[4]{\frac{1}{3}}(1-\sqrt{2})$.
257. $a\sqrt{2}+a\sqrt{-2}$.
258. $2a^2+2a^2\sqrt{-1}$.
259. $\sqrt{2}-1$.
260. $1+\sqrt{2}$.
261. $\frac{1}{2}a-2b^{\frac{1}{2}}$.
262. $\frac{x^{\frac{1}{2}}}{a^{\frac{1}{2}}}-1+\frac{a^{\frac{1}{2}}}{x^{\frac{1}{2}}}$.
263. $x^{\frac{m}{n}}y^{-\frac{n}{m}}-x^{-\frac{n}{m}}y^{\frac{m}{n}}$.
264. $xy^{-\frac{1}{3}}-x^{-\frac{1}{2}}y^{\frac{2}{3}}$.
265. $\frac{\sqrt[5]{x^3}}{2}-5\sqrt[5]{y^4}$.
266. $\cdot5$.
267. $-\frac{9+2\sqrt{2}+3\sqrt{3}+2\sqrt{6}}{19}$.
268. $\frac{2\sqrt{2}+\sqrt{3}}{5}$.
269. $\sqrt{5}+\sqrt{2}$.
270. $9+2\sqrt{15}$.
271. $\frac{4\sqrt{5}-3\sqrt{2}}{2}$.
272. $\frac{8+3\sqrt{2}+2\sqrt{3}+\sqrt{6}}{8}$.
273. $\frac{3\sqrt{2}+2\sqrt{3}-\sqrt{30}}{12}$.

274. $\dfrac{9\sqrt{3}-9\sqrt[3]{5}+3\sqrt[6]{16875}-15+5\sqrt[6]{675}-5\sqrt[3]{25}}{2}$.

275. $\dfrac{5-8\sqrt[3]{3}+4\sqrt[3]{9}}{11}$. 276. $\dfrac{5+8\sqrt[3]{3}+4\sqrt[3]{9}}{5}$.

277. $\dfrac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{a-b}$.

278. $\dfrac{\sqrt{3^{15}}+3^6\cdot\sqrt[3]{5^2}+\sqrt{3^9}\cdot\sqrt[3]{5^4}+675+\sqrt{3^3}\cdot\sqrt[3]{5^8}+\sqrt[3]{5^{10}}}{19058}$.

279. $17-12\sqrt{2}+12\sqrt[3]{3}+8\sqrt[3]{9}-26\sqrt[6]{72}$.

280. $a-b$. 281. b.

282. For $(a+b+c)^2>4bc$. 283. $\dfrac{1}{2}\left(xy+\dfrac{1}{xy}\right)$.

284. $\left(\dfrac{2}{3}\right)^{\frac{2}{3}}$. 285. $\sqrt[5]{2}$.

286. $\sqrt[3]{9}$. 287. $\sqrt{2}+\sqrt{7}$.

288. $\sqrt{10}+\sqrt{7}$. 289. $\sqrt[3]{2}+\sqrt[3]{5}$.

290. $a+\sqrt{a}$; and $a^{\frac{3}{2}}-1$.

291. $\sqrt[3]{3}$ is the greatest; $\sqrt[5]{5}$ the least.

294. $\sqrt{\dfrac{1}{2}\{1+\sqrt{-1}\}}$. 295. $\dfrac{ac+bd}{c^2+d^2}+\dfrac{bc-ad}{c^2+d^2}\sqrt{-1}$.

296. The former is possible, the latter impossible.

297. $\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}\sqrt{-1}$. 298. $x=ac+bd$; $y=ad-bc$.

299. $a-d+(b-c)\sqrt{-1}$, and $a-d-(b-c)\sqrt{-1}$.

301. $x=\dfrac{1}{2}$, $y=\dfrac{1}{2}$.

EQUATIONS.

302. $x=\dfrac{a+2c}{b-1}$. 303. $x=\dfrac{abc}{a+b}$.

304. $x=8$. 305. $x=9$.

306. $x=6$. 307. $x=1$.

308. $x=-4$. 309. $x=2$.

310. $x=\dfrac{bf(h-g)}{af+2bc-bfq}$. 311. $x=120$.

312. $x=13$. 313. $x=\dfrac{2}{3}$.

314. $x=5$. 315. $x=4$.

316. $x=42$.
317. $x=12$.
318. $x=1\frac{13}{67}$.
319. $x=12$.
320. $x=-8$.
321. $x=9$.
322. $x=\frac{73}{68}$.
323. $x=5.1$.
324. $x=\frac{1}{ab}$.
325. $x=\frac{1}{2}$.
326. $x=25a+24b$.
327. $x=19$.
328. $x=72$.
329. $x=8$.
330. $x=2$.
331. $x=11$.
332. $x=b\sqrt{1-\frac{2a}{bc}}$.
333. $x=\frac{3}{5}$.
334. $x=8$.
335. $x=-107$.
336. $x=28$.
337. $x=4$.
338. $x=4\frac{1}{3}$.
339. $x=4$.
340. $x=\frac{9}{20}$.
341. $x=\left(\frac{83}{30}\right)^2$.
342. $x=\frac{b^2-a^2}{4a-b}$.
343. $x=3$.
344. $x=\frac{\sqrt{a}}{\sqrt{a}+2}$.
345. $x=\frac{14}{17}$.
346. $x=\frac{b}{2}\left(\frac{b-2a}{b-a}\right)$.
347. $x=\frac{9a}{16}$.
348. $x=\frac{64a}{1025}$.
349. $x=\frac{a}{2}\sqrt{\frac{a^2-4}{a^2-1}}$.
350. $x=\frac{ab^{\frac{2}{3}}}{a^2-b^{\frac{2}{3}}}$.
351. $x=\frac{16}{25}$.
352. $x=\frac{24}{25}$.
353. $x=a\frac{n-1}{\sqrt{2n-1}}$.
354. $x=\frac{1}{4}\left(1-a+\frac{1}{1-a}\right)^2$.
355. $x=n\sqrt{a-\frac{n^2}{4}}$.
356. $x=\frac{a^2-b^2}{6a}$.
357. $x=1$.
358. $x=\frac{961a}{1025}$.
359. $x=\frac{\sqrt{3}}{2}a$.
360. $x=\frac{1}{2}\left\{\sqrt[3]{a}+\frac{1}{\sqrt[3]{a}}\right\}-1$.
361. $x=-a$.
362. $x=\sqrt{\frac{a-2}{a+4}}$.
363. $x=2\sqrt{(1-a)(1-\frac{1}{3}a)}$.

364. $x=\frac{1}{12}\cdot\frac{3a-1}{a+1}$.

365. $x=2\sqrt{a}+a^{-1}$.

366. $x=\sqrt[4]{\frac{\sqrt{2}+1}{2}}$.

367. $x=a\left\{1-\left(\frac{2\sqrt{b}}{1+b}\right)^4\right\}$.

368. $x=\frac{4}{a}\sqrt{1-\frac{4}{a^2}}$.

369. $x=\frac{1}{a}\sqrt{\frac{2a}{b}-1}$.

---

370. $x=\frac{bc}{a+b}$, $y=\frac{ac}{a+b}$.

371. $x=1$, $y=-1$.

372. $x=7$, $y=2$.

373. $x=\frac{ac+b^2}{a^2+b}$, $y=\frac{ab-c}{a^2+b}$.

374. $x=5$, $y=2$.

375. $x=\frac{2}{3}$, $y=\frac{14}{9}$.

376. $x=18$, $y=10$.

377. $x=144$, $y=216$.

378. $x=\frac{m^2-n^2}{ma-nb}$, $y=\frac{m^2-n^2}{mb-na}$.

379. $x=2\frac{62}{65}$, $y=1\frac{29}{35}$.

380. $x=\frac{1}{4}$, $y=\frac{1}{5}$.

381. $x=\cdot 02$, $y=2\cdot 9$.

382. $x=99$, $y=15$.

383. $x=7$, $y=9$.

384. $x=5$, $y=5$.

385. $x=10$, $y=5$.

386. $x=21$, $y=20$.

387. $x=18$, $y=24$.

388. $x=3$, $y=2$.

389. $x=\frac{abc\,(ab+ac-bc)}{a^2b^2+a^2c^2-b^2c^2}$, $y=\frac{abc\,(ac-ab-bc)}{a^2b^2+a^2c^2-b^2c^2}$.

390. $x=a+b\pm 2\sqrt{ab}$
$y=a+b\mp 2\sqrt{ab}$.

391. $x=\pm\sqrt{\frac{a+b}{a-b}}$, $y=\pm\sqrt{\frac{a-b}{a+b}}$.

392. $x=16$, $y=25$.

393. $x=\frac{ab}{a-b}$, $y=\frac{ab}{a+b}$.

394. $x=7$, $y=10$, $z=9$.

395. $x=1$, $y=2$, $z=4$.

396. $x=5$, $y=6$, $z=7$.

397. $x=2$, $y=4$, $z=6$.

398. $x=5$, $y=7$, $z=-3$.

399. $x=\frac{12}{7}$, $y=\frac{12}{5}$, $z=-12$.

400. $x=\frac{2}{a+b-c}$, $y=\frac{2}{a-b+c}$,
$z=\frac{2}{b+c-a}$.

401. $x=-6\frac{6}{23}$, $y=2\frac{2}{71}$,
$z=3\frac{21}{41}$.

402. $x=\frac{1}{2}$, $y=\frac{1}{3}$, $z=\frac{1}{4}$.

403. $x=6$, $y=1$, $z=3$.

404. $x=18$, $y=12$, $z=4$.

405. $x=\frac{1}{(a-c)(b-c)}$, $y=\frac{1}{(a-b)(b-c)}$, $z=\frac{1}{(a-c)(a-b)}$.

406. $x=1$, $y=4$, $z=27$.

407. $x=1$, $y=2$, $z=3$.

408. $x=3$, $y=5$, $z=7$.

409. $x=3$, $y=7$, $z=11$, $w=20$.

410. $x=5$, $y=4$, $z=3$, $u=2$, $t=1$.

---

411. $x=12, -2$.

412. $x=a, b$.

413. $x=6, -4\tfrac{1}{2}$.

414. $x=\pm\sqrt{\frac{24}{89}}$.

415. $x=\sqrt{\frac{607\pm\sqrt{238297}}{174}}$.

416. $x=\frac{1}{2}(5\pm\sqrt{22{\cdot}6})$.

417. $x=107, -106$.

418. $x=\pm\sqrt{\frac{55}{62}}$.

419. $x=\pm\sqrt{30{\cdot}05}$.

420. $x=136, -25$.

421. $x=\frac{3}{2}, -\frac{15}{22}$.

422. $x=5, -4$.

423. $x=80, -75$.

424. $x=\frac{1}{2}, -\frac{4}{3}$.

425. $x=\pm 8$.

426. $x=9\tfrac{15}{17}, -11$.

427. $x=2, 4$.

428. $x=7, -\frac{1}{3}$.

429. $x=12\tfrac{1}{12}, -12$.

430. $x=\frac{14\pm 2\sqrt{34}}{15}$.

431. $x=2, \frac{1}{2}$.

432. $x=\frac{83\pm\sqrt{18109}}{34}$.

433. $x=8\pm\sqrt{601}$.

434. $x=\frac{\pm 2\sqrt{277}-33}{19}$.

435. $x=\pm\sqrt{\frac{2149}{1355}}$.

436. $x=3, -8{\cdot}7$.

437. $x=a, -b$.

438. $x=\frac{3}{4}a, \frac{1}{2}a$.

439. $x=\pm a\sqrt{\pm 8\sqrt{2}-11}$.

440. $x=\pm\frac{\sqrt{9493}-35}{12}$.

441. $x=4, -1$.

442. $x=\pm 5, \pm 3\sqrt{2}$.

443. $x=1, 1\pm 2\sqrt{15}$.

444. $x=3, -\frac{1}{2}, \frac{1}{4}(5\pm\sqrt{1329})$.

445. $x=4,\ -5.$

446. $x=a,\ 0,\ \frac{a}{2}\left(1\pm\sqrt{5-\frac{8b}{a}}\right).$

447. $x=5,\ -6,\ \frac{1}{2}(\pm\sqrt{377}-1).$

448. $x=\frac{1}{2}(3\pm\sqrt{5}),\ \frac{1}{6}(9\pm\sqrt{-83}).$

449. $x=6,\ -1,\ \frac{1}{2}(5\pm 3\sqrt{-3}).$

450. $x=\frac{n}{q},\ -\frac{p}{m}.$

451. $x=6,\ -\frac{2}{3}.$

452. $x=\frac{5}{2},\ \frac{5}{8}.$

453. $x=\pm 5,\ \pm 4\sqrt{2}.$

454. $x=4,\ -2,\ -1\pm\sqrt{7}.$

455. $x=\frac{1}{2}(3\pm\sqrt{29}),\ \frac{1}{2}(-3\pm\sqrt{21}).$

456. $x=\cdot 04,\ 4.$

457. $x=\frac{28}{27}a.$

458. $x=\pm\sqrt{a^2-\left(h^2\pm\sqrt{a+\frac{h^4}{2}}\right)^4}.$

459. $x=\frac{63}{65}a,\ 0.$

460. $x=\pm\sqrt{-\frac{1}{3}}.$

461. $x=a\left(\frac{2^{\pm m}-1}{2^{\pm m}+1}\right)^{\frac{1}{n}}.$

462. $x=\frac{1}{2}(7\pm\sqrt{13}).$

463. $x=\frac{3a}{32}(3\pm\sqrt{21}).$

464. $x=\sqrt[n]{\frac{2ac}{b(c^2+1)}}.$

465. $x=\frac{4}{81}.$

466. $x=0,\ \pm\sqrt{2ab-b^2}.$

467. $x=\pm\frac{2}{b\sqrt{4a-b^2}}.$

468. $x=a,\ \frac{a}{8}(-3\pm\sqrt{-7}).$

469. $x=\left\{\frac{a^{\frac{r}{s}}+1}{a^{\frac{r}{s}}-1}\right\}^{\frac{1}{(m+n)^2}}.$

470. $x=\left(\frac{a+b}{a-b}\right)^{\pm\frac{2pq}{p-q}}.$

471. $x=4,\ 8.$

472. $x=3^{-\frac{2}{3}},\ 1.$

473. $x=\frac{1}{a^2},\ \frac{1}{4a^2}.$

474. $x=4,\ (-7)^{\frac{2}{3}}.$

475. $x=1,\ 16,\ \frac{1}{2}(1\pm 3\sqrt{-7}).$

476. $x=4,\ 9,\ \frac{1}{2}(-13\pm\sqrt{-27})$.

477. $x=3,\ -\frac{3}{4},\ \frac{9}{8}(9\pm\sqrt{97})$. 478. $x=8,\ 29\mp 7\sqrt{-10}$.

479. $x=-1,\ \frac{1}{2}(1\pm\sqrt{4p-3})$.

480. $x=-3,\ \frac{1}{2}(3\pm\sqrt{-3})$. 481. $x=4,\ 1\pm\sqrt{-1}$.

482. $x=1,\ \frac{1}{3}(6\pm\sqrt{-3})$.

483. $x=2\alpha(\alpha\pm\sqrt{\alpha^2+a})$, where $\alpha=\frac{1}{4}(-a\pm\sqrt{a^2+4b})$.

484. $x=\pm\sqrt{\frac{3\pm\sqrt{41}}{2}},\ \pm\sqrt{\frac{1\pm\sqrt{37}}{2}}$.

485. $x=2,\ -1$.

486. $x=-1,\ \frac{1}{2}(1-p\pm\sqrt{p^2-2p-3})$.

487. $x=1,\ \frac{1}{4}(-1\pm\sqrt{-7})$.

488. $x=\pm\frac{1}{a}\sqrt{\left\{2(-1\pm\sqrt{1-a^4})\pm\sqrt{2(1\pm\sqrt{1-a^4})}\right\}}$.

489. $x=\sqrt{\left\{\frac{2}{a}-\frac{1}{2}\pm\sqrt{\left(\frac{2}{a}\right)^2-\frac{3}{4}}\right\}}$.

490. $x=\sqrt{\left\{\frac{c+4\pm 2\sqrt{3(c+1)}}{c-2}\right\}}$.

491. $x=4,\ 6\frac{1}{4},\ \frac{1}{8}(209-13\sqrt{249})$.

492. $x=4,\ -3,\ \frac{1}{2}(1\pm\sqrt{-43})$.

493. $x=5,\ -1,\ 2\pm\sqrt{5}$.

494. $x=\pm 3,\ \frac{1}{6}(-13\pm\sqrt{-155})$.

495. $x=1,\ \frac{1}{2}(-3\pm\sqrt{5})$.

496. $x=\frac{1}{4}(3\pm\sqrt{-7}),\ \frac{1}{4}(-1\pm\sqrt{-15})$.

497. $x=-1,\ 2,\ \frac{1}{2}(1\pm\sqrt{-3}),\ -1\pm\sqrt{-3}.$

498. $x=\frac{1+2h\pm\sqrt{3(4h-1)}}{2(1-h)}.$

499. $x=1\pm\sqrt{3}\pm\sqrt{3\pm2\sqrt{3}}.$

499*. $x=\frac{1}{2}(r\pm\sqrt{r^2-4})$, where $r=\frac{1+4a+\sqrt{5(1+4a)}}{2(1-a)}.$

$\frac{1}{2}(s\pm\sqrt{s^2-4})$, and $s=\frac{1+4a-\sqrt{5(1+4a)}}{2(1-a)}.$

---

500. $x=6,\ -4,$
$y=2,\ -3.$

501. $x=5,\ -4,$
$y=4,\ -5.$

502. $x=25\pm\sqrt{345},$
$y=20\pm\sqrt{345}.$

503. $x=3,\ 2\frac{3}{17},$
$y=2\frac{5}{17},\ 1.$

504. $x=9,\ -8\frac{1}{2},$
$y=-8,\ 9\frac{1}{2}.$

505. $x=3{\cdot}5,\ -3\frac{32}{67},$
$y=4{\cdot}25,\ -2\frac{66}{67}.$

506. $x=8,\ -{\cdot}5,$
$y=6,\ -2{\cdot}5.$

507. $x=3,\ -1\frac{26}{27},$
$y=-4,\ 8\frac{11}{27}.$

508. $x=5,\ 1{\cdot}7,$
$y=3,\ -{\cdot}3.$

509. $x=21\frac{3}{28},$
$y=6\frac{27}{28}.$

510. $x=\pm7,\ \pm6,$
$y=\pm6,\ \pm7.$

511. $x=8,\ -8{\cdot}2,$
$y=12,\ -12{\cdot}3.$

512. $x=16,\ 4,$
$y=4,\ 16.$

513. $x=27,\ 8,$
$y=8,\ 27.$

514. $x=4,\ -1,$
$y=1,\ -4.$

515. $x=9,\ 1,$
$y=1,\ 9.$

516. $x=5,\ 3,$
$y=3,\ 5.$

517. $x=625,\ 1,$
$y=1,\ 3125.$

518. $x=4,\ 1,\ \frac{1}{2}(5\pm\sqrt{-159}),$
$y=1,\ 4,\ \frac{1}{2}(5\mp\sqrt{-159}).$

519. $x=11,\ -52,$
$y=3,\ -60.$

520. $x=\frac{1}{2}\left\{b\pm\sqrt{-\left(b^2\pm2\sqrt{\frac{4a^5+b^5}{5b}}\right)}\right\},$

$y=\frac{1}{2}\left\{b\mp\sqrt{-\left(b^2\pm2\sqrt{\frac{4a^5+b^5}{5b}}\right)}\right\}.$

521. $x=3,\ -2,\ \frac{1}{2}\left\{\pm\sqrt{-\frac{79}{3}}+1\right\},$

$y=2,\ -3,\ \frac{1}{2}\left\{\pm\sqrt{-\frac{79}{3}}-1\right\}.$

522. $x=81,\ 16,\ \frac{1}{3}(7-36\sqrt{-22}),$

$y=16,\ 81,\ \frac{1}{3}(7+36\sqrt{-22}).$

523. $x=16,\ 735\frac{1}{81},\ y=4,\ 723\frac{1}{81}.$

524. $x=\pm 3,\ \pm\sqrt{6},\ \pm\sqrt{\frac{15\pm 3\sqrt{5}}{2}},$

$y=2,\ -1,\ \frac{1}{2}(1\pm\sqrt{-47}),\ \frac{1}{2}(1\pm 3\sqrt{5}).$

525. $x=3,\ 2,\ 5,\ 1,$
$y=2,\ 3,\ 1,\ 5.$

526. $x=7,\ -1,\ \ 3\pm\sqrt{2},$
$y=1,\ -7,\ -3\pm\sqrt{2}.$

527. $x=\pm 5,\ \pm\sqrt{\frac{1}{2}(\pm 10\sqrt{-2}\pm\sqrt{1482})},$

$y=\pm 2,\ \pm\sqrt{\frac{1}{2}(\pm 10\sqrt{-2}\mp\sqrt{1482})}.$

528. $x=4,\ 2,\ \frac{2}{3}(-2\pm\sqrt{-14}),$

$y=2,\ 4,\ \frac{2}{3}(-2\mp\sqrt{-14}).$

529. $x=\pm 2,\ \pm\frac{4}{3}\sqrt{3},$

$y=\pm 6,\ \pm\frac{10}{3}\sqrt{3}.$

530. $x=\pm\frac{1}{2}\sqrt{3a^2+b^2\mp\sqrt{a^4+6a^2b^2+b^4}},$

$y=\pm\frac{1}{2}\sqrt{a^2+3b^2\pm\sqrt{a^4+6a^2b^2+b^4}}.$

531. $x=4,\ 1,$
$y=8.$

532. $x=5,\ 18\frac{1}{8},$
$y=4,\ 17\frac{7}{8}.$

533. $x=\frac{ab}{b^2-a^2}(b\mp\sqrt{2a^2-b^2}),$

$y=\frac{ab}{b^2-a^2}(b\pm\sqrt{2a^2-b^2}).$

534. $x=\frac{1}{4}(1\pm\sqrt{2}),$
$y=\frac{1}{4}.$

535. $x=8,\ -4,\ 8(19\mp 8\sqrt{6}),$
$y=4,\ \ 1,\ 8\ (5\mp 2\sqrt{6}).$

536. $x=0,\ 4,$
$y=0,\ 3.$

537. $x=\pm 5,$
$y=\pm 4.$

538. $x=\sqrt[3]{2},\ y=\sqrt[3]{2}.$

539. $x=\frac{1}{4b}\left\{a^2+b^2\pm\sqrt{10a^2b^2-3a^4-3b^4}\right\},$

$y=\frac{1}{4b}\left\{a^2+b^2\mp\sqrt{10a^2b^2-3a^4-3b^4}\right\}.$

540. $x=3,\ -2,\ \frac{1}{2}\left\{13^{\frac{2}{3}}\pm\sqrt{-11\sqrt[3]{13}}\right\},$

$y=-2,\ 3,\ \frac{1}{2}\left\{13^{\frac{2}{3}}\mp\sqrt{-11\sqrt[3]{13}}\right\}.$

541. $x=\frac{4}{3}\sqrt[3]{\frac{100}{3}},\ y=\frac{1}{9}.$

542. $x=2,\ -1,\ \mp\sqrt{-2},$
$y=1,\ \pm\frac{1}{4}(\pm\sqrt{6}-\sqrt{-2}).$

543. $x=\pm 2,\ \pm\sqrt{-1},\ \pm\sqrt{\pm\sqrt{-13}+1{\cdot}5},$
$y=\pm 1,\ \pm 2\sqrt{-1},\ \pm\sqrt{\pm\sqrt{-13}-1{\cdot}5}.$

544. $x=\pm\frac{\sqrt[5]{10}}{2\sqrt{5}}(\sqrt{5}+1),$
$z=\pm\frac{\sqrt[5]{10}}{2\sqrt{5}}(\sqrt{5}-1).$

545. $x=\pm\sqrt[5]{\frac{9216}{735}},$
$y=\pm\sqrt[4]{48\sqrt{\frac{735}{9216}}}.$

546. $x=\pm\sqrt{\pm ac},\ \frac{1}{2}\left\{a-b+c\pm\sqrt{(a-b+c)^2-4ac}\right\},$

$y=\pm\sqrt{\pm bc},\ \frac{1}{2}\left\{b+c-a\mp\sqrt{(a-b+c)^2-4ac}\right\}.$

547. $x=\pm\sqrt{na},\ \frac{1}{2}(n\pm\sqrt{n^2\mp 4n\sqrt{ab}}),$

$y=\pm\sqrt{nb},\ \frac{1}{2}(n\mp\sqrt{n^2\mp 4n\sqrt{ab}}).$

548. $x=5,\ -4{\cdot}6,\ \frac{25\pm 16\sqrt{78}}{29},$

$y=3,\ -2{\cdot}76,\ \frac{21(25\pm 16\sqrt{78})}{(29)^2}.$

549. $x=\pm\sqrt{\frac{25\pm 5\sqrt{41}}{8}},\ y=\pm\sqrt{\frac{25\mp 5\sqrt{41}}{8}}.$

550. $x=12,\ -9{\cdot}5,\ \frac{1}{4}(-45\pm\sqrt{3849}),$

$y=2,\ -\frac{19}{12},\ \frac{3}{8}(45\mp\sqrt{3849}).$

551. $x=\pm\frac{1}{2}\left\{(4\pm\sqrt{6})\pm\sqrt{18\pm8\sqrt{6}}\right\},$

$y=\pm\frac{1}{2}\left\{(2\pm\sqrt{6})\pm\sqrt{18\pm8\sqrt{6}}\right\}.$

552. $x=4,\ 2,\ \frac{1}{6}(-35\pm\sqrt{-5635}),$

$y=2,\ 4,\ \frac{1}{6}(-35\mp\sqrt{-5635}).$

553. $x=\pm4,\ \pm4\sqrt{-1},\ \pm1,\ \pm\sqrt{-1},$
$y=\pm1,\ \pm\sqrt{-1},\ \pm4,\ \pm4\sqrt{-1}.$

554. $x=\frac{3}{2}(19\pm\sqrt{105}),$

$y=\frac{1}{6}(3\pm\sqrt{105}).$

555. $x=\pm4,\ \pm\frac{10}{3}\sqrt{3},$
$y=\pm3,\ \pm\sqrt{3},$
$z=\pm2,\ \mp\frac{8}{3}\sqrt{3}.$

556. $x=\left(-\frac{1}{2}\pm\frac{1}{6}\sqrt{57}\right)^8,\ \left(-\frac{1}{2}\pm\frac{1}{6}\sqrt{-39}\right)^8,$

$y=\left(-\frac{1}{2}\pm\frac{1}{6}\sqrt{57}\right)^4,\ \left(-\frac{1}{2}\pm\frac{1}{6}\sqrt{-39}\right)^4.$

557. $x=\pm\sqrt{\frac{a^{\alpha^2}}{b^{\beta^2}}},\ y=\pm\sqrt{\frac{b^{2\alpha\beta-\beta^2}}{a^{2\alpha\beta-\alpha^2}}}.$

558. $x^2=-\frac{2592}{1247},\ y^2=-\frac{137600}{1349},\ z^2=-\frac{349^2\times29}{288\times43}.$

560. $x^2-2x-15=0.$

561. $x^3-8x^2+x+42=0.$

562. $x^4+10x^3-19x^2-480x-1392=0.$

563. $9x^5+6x^4-439x^3-294x^2-98x=0.$

565. $h^2b\mp h^2\sqrt{b^2-4a^2c}=a^2k\mp a^2\sqrt{k^2-4h^2t}.$

566. $ax^2+by^2+cz^2=d^2(\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c})^3.$

567. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{2x}+\sqrt{3y}}=\frac{a(\sqrt{3}-\sqrt{2})+2}{2a\sqrt{6}-(\sqrt{3}-\sqrt{2})}.$

PROBLEMS IN SIMPLE EQUATIONS.

568. 9.
569. 14.
570. 12.
571. 12.
572. At $104\frac{8}{73}$ miles from London, and in $10\frac{70}{73}$ hours.
573. £8 15*s*.
574. 8 hours.
575. 1000 men.
576. $10\frac{16}{53}$ days, and $45\frac{1}{2}$ days.
577. $17\frac{5}{36}$, $14\frac{17}{36}$, and $12\frac{5}{36}$ gallons.
578. £1000, £1500, £2250, and £3250.
579. $\frac{4}{15}$.
580. $2\frac{20}{53}$ days.
581. 1080 yds., and $16\frac{1}{2}$ min.
582. 3 miles, 3 yards.
583. £25.
584. 6 hours, and 3 hours.
585. 21.
586. 240 acres; 28*s*. an acre.
587. 5 and 6.
588. A with £70; B with £130.
589. 28 bushels of barley; 20 of rye; and 52 of wheat.
590. The man in $21\frac{3}{7}$, the woman in 50 days.
591. 65.
592. The weight is 50 lbs.; and the prime cost 28*s*. per lb.
593. The times are as $(m+1)n:(n+1)m:\frac{mn(m+1)(n+1)}{p\{m(n+1)+n(m+1)\}}$.

PROBLEMS IN QUADRATIC EQUATIONS.

594. ± 18, and ± 14.
595. ± 12, ± 16, and ± 18.
596. 10 and 6.
597. ± 9, and ± 4.
598. 18 sheep.
599. 4 and 2.
600. A relieved 120, and B relieved 80.
601. 4550.
602. 64 gallons at the 1st, 48 at the 2nd, 36 at the 3rd, and 27 at the 4th draught.
603. 24.
604. 12 and 5.
605. 8 and 7.
606. The fraction is $\frac{2}{1}$.
607. $\frac{\sqrt{5}}{2}$, $\frac{1}{2}$, and $\frac{3}{2}$.
608. 25 miles from M.
609. 90, 120, and 150 yards.
610. The sherry £2 a dozen, the claret £3 a dozen.
611. 25, 13, and 6.
612. 12 companies.
613. 693 men.
614. 3 and 2.
615. 4 yards, and 5 yards.
616. 1 : 9 in the first mixture, and 1 : 4 in the second.

RATIO, PROPORTION, AND VARIATION.

618. $a^4-b^4 : a^2-b^2$ is the greater if a and b are integers.
619. $a^3+b^3 : a^2+b^2$ is the greater ratio.
626. $xy=\frac{12}{25}(x^2+y^2)$.
627. $y^2=\frac{b^2}{a^2}(x^2-a^2)$.
628. $y^2=4ax$.
630. 1 : 2 is the ratio.
631. 2 gallons from the 1st, and 14 from the second.
632. The value of the diamond is £$\frac{mcx^2}{(m+1)a^2}$, and of the ruby is £$\frac{cx^{\frac{3}{2}}}{(m+1)b^{\frac{3}{2}}}$.

ARITHMETICAL PROGRESSION.

633. n^2, and 400.
634. 25452.
635. $40\frac{1}{2}$.
636. 0.
637. 204.
638. $\frac{n}{3}(40-n)$, and 0.
639. 0.
640. 0.
641. 407.
642. $20\frac{5}{12}$.
643. $-1627\frac{2}{9}$.
644. $\frac{n-1}{2}$.
645. $n\{(a^2+x^2)-(n-3)ax\}$.
646. $\frac{n}{a+b}\left\{na-\frac{n+1}{2}b\right\}$.
647. 19 or 18.
648. 13 or 7.
649. 4.
650. 60.
651. $b=3$.
652. 19.
653. The series is $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}$, &c.
654. 4.
655. 4.
656. The ratio of their 1st terms is $\frac{2}{3}$, and of their 2nd $-\frac{4}{5}$.
657. The ratio is $\frac{1}{3}$.
658. $\frac{p-q}{ab}+\frac{r-p}{ac}+\frac{q-r}{bc}=0$.
659. The 1st term is $\frac{Q(p-1)-P(q-1)}{p-q}$,

the sum $=\frac{Q(2p-n-1)-P(2q-n-1)}{p-q}\cdot\frac{n}{2}$.

660. The series is a, $a+\frac{l-a}{n-1}$, $a+2\frac{l-a}{n-1}$, &c.

661. $\frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}, 0, -\frac{1}{5}, -\frac{2}{5}, -\frac{4}{5}$.

662. $-2, -6, -10, -14$. 663. 207, 297, 387.

664. $n=14$.

665. $\frac{a\{(r+1)(n-p+1)+m(p-q)\}+b\{(r+1)p-m(p-q)\}}{(r+1)(n+1)}$.

666. $m=8$. 668. pth term is $a+(p-1)d$,

669. $\frac{1}{2}nr(nr+1)$. qth is $m-(m-n)\frac{p}{2q}$.

670. $n(3n-1)\frac{a}{2}+n(n-1)(7n-2)\frac{d}{1.2.3}$.

671. 25 terms.

672. The sum is £135 4*s.*; the last payment £5 3*s.*

673. AB $=7$, BC $=9$, CD $=11$, DE $=13$, and EF $=15$ miles.

674. 8 days. 675. In 2, 3, 4, 5 and 6 days.

676. 5*s.*, 7*s.*, and 9*s.*

GEOMETRICAL PROGRESSION.

677. 9841. 678. $39\frac{9}{16}$.

679. $-\frac{1261}{384}$. 680. $\frac{1}{3}\{1-(-2)^n\}$.

681. $-\frac{203}{40}(\sqrt{10}-2)$. 682. $-(\cdot 6)^{\frac{1}{2}} \times 1111(\sqrt{10}-1)$.

683. $\frac{3}{25}$. 684. $5\frac{394}{729}$, and $5\frac{2}{5}$.

685. $\frac{8}{21}$. 686. $\frac{5}{7}$.

687. 16. 688. $2\frac{7}{9}$.

689. $\frac{4}{33}$. 690. $\frac{1-x}{1+x}$.

691. $10\frac{1}{8}$. 692. $4+3\sqrt{2}$.

693. $\frac{f+x}{g+x}$.

694. $\frac{x^{2n}-y^{2n}}{(x+y)x^{2n-2}}$, and $\frac{x^2}{x+y}$; the ratio is $x^{2n}-y^{2n} : x^{2n}$.

695. $\frac{(3a)^{\frac{n}{2}}-(2x)^{\frac{n}{2}}}{(3a)^{\frac{n-3}{2}}.x\sqrt{6}\{(3a)^{\frac{1}{2}}-(2x)^{\frac{1}{2}}\}}$, and $\frac{3a^{\frac{3}{2}}}{x\sqrt{2}\{(3a)^{\frac{1}{2}}-(2x)^{\frac{1}{2}}\}}$.

696. $\frac{4-2\sqrt{-1}}{15}$. 697. $\frac{1-x}{(1+x)^2}$.

698. $\left(\frac{1+x}{1-x}\right)^2$.

699. 3.

700. 1.

701. $4(2^n-1)-3n$.

702. $2(2^n-1)-n$.

703. $2(2^n-1)+\frac{n}{2}(n+1)$.

704. $5n-\frac{9}{2}\left(1-\frac{1}{3^n}\right)$.

705. $\frac{5}{24}$.

706. $(100)^{\frac{1}{3}}$, and $(100)^{\frac{2}{3}}$.

707. 2, 8, and 32.

708. 4, 8, 16, and 32.

709. 49 and 1.

710. $a:b=2+\sqrt{3}:2-\sqrt{3}$.

711. Sum $=\frac{1}{b^{2n-2}}\cdot\frac{a^{2n}-b^{2n}}{(a^2-b^2)^2}$.

712. The pth term is $\sqrt{mn}$,
the qth is $m\left(\frac{n}{m}\right)^{\frac{p}{2q}}$.

713. $a^2+b^2+c^2$ is the greater.

714. $S_1:S_2=27:1$.

716. The nth term is $\left(\frac{P^{n-q}}{Q^{n-p}}\right)^{\frac{1}{p-q}}$.

718. The rth mean is $\frac{s_2^{2n}}{s_1}\left\{\frac{r}{m+1}\left(\frac{s_1}{s_2}\right)^q+\frac{m-r+1}{m+1}\left(\frac{s_1}{s_2}\right)^p\right\}\cdot\frac{s_1^2-s_n^2}{s_1^{2n}-s_2^{2n}}$.

720. £108, £144, £192, and £256.

HARMONICAL PROGRESSION.

721. $\frac{6}{5}$, $\frac{3}{2}$, 2, 3, ∞, -6; and 2, $\frac{12}{5}$, 3, 4, 6, 12, ∞. This series can be continued indefinitely either way.

722. $\frac{12}{5}$, and 3.

723. 8, and 12.

724. $\frac{12}{5}$, 3, 4, and 6.

725. $\frac{140}{121}$, $\frac{70}{51}$, $\frac{140}{83}$, $\frac{35}{16}$, $\frac{28}{9}$, and $\frac{70}{13}$.

726. $\frac{(n+1)xy}{ny+x}$, $\frac{(n+1)xy}{(n-1)y+2x}$, ... $\frac{(n+1)xy}{y+nx}$.

727. $\frac{(n-1)ab}{(n-2)y+x}$, $\frac{(n-1)ab}{(n-3)y+2x}$, ... $\frac{(n-1)ab}{y+(n-2)x}$.

728. The series is x, y, $\frac{xy}{2x-y}$, $\frac{xy}{3x-2y}$...... $\frac{xy}{(n-1)x-(n-2)y}$.

730. a^2+c^2 is the greater.

731. $\frac{(m-n)MN}{mN-nM}$.

733. $y=\dfrac{2(a+b)}{\left\{\left(\dfrac{a}{b}\right)^{\frac{1}{4}}+\left(\dfrac{b}{a}\right)^{\frac{1}{4}}\right\}^{2}}$.

736. If a, b, c be the 3 sides, $\dfrac{b}{a}=\pm\sqrt[4]{2}$; $\dfrac{c}{a}=\pm\sqrt{1\pm\sqrt{2}}$.

737. 2, 3, and 6.

THE PILING OF BALLS AND SHELLS.

739. 8436.
740. 11440.
741. 24395.
742. 2561.
743. 12859.
744. 25707.
745. 816 in triangular, 1496 in square pile.
746. 1040 in rectangular, 650 in square pile.
747. 1512.

PERMUTATIONS AND COMBINATIONS.

748. 56.
749. 255.
750. 14 ways.
751. 238170297200 nights; on 26794158435 of which a different man would be posted.
752. (1) 19958400, (2) 34650, (3) 3991680, (4) 180, (5) 3326400, (6) 92400.
753. 13860.
754. 495; in 9 of which c will occur.
755. 52 cards.
756. $n=12$.
757. $n=7$.
758. $n=12$.
759. $n=15$, $r=6$.
761. 116280 words.
762. 27907200 words.
763. 4080 words.
764. $n=6$.
765. 1728 throws.
766. $n=12$.
767. n^r.
768. $(p+1)(q+1)(r+1)(\&c.)-1$.
769. $r=n$.
770. $C_{n+1}^{(r)}=C_{n}^{(r)}+C_{n}^{(r-1)}$.
771. The number of homogeneous products is expressed by

$$\frac{n(n+1)(n+2)\ldots(n+r-1)}{1.2.3\ldots r};$$

and the required number of terms is 286.

BINOMIAL AND MULTINOMIAL THEOREMS.

772. (1) $a^9+9a^8x+36a^7x^2+84a^6x^3+126a^5x^4+126a^4x^5$
$+84a^3x^6+36a^2x^7+9ax^8+x^9.$

(2) $a^7-7a^6x+21a^5x^2-35a^4x^3+35a^3x^4-21a^2x^5+7ax^6$
$-x^7.$

(3) $16x^4-96x^3y+216x^2y^2-216xy^3+81y^4.$

(4) $15625-3125x+\frac{3125}{12}x^2-\frac{625}{54}x^3+\frac{125}{432}x^4-\frac{5}{1296}x^5$
$+\frac{1}{46656}x^6.$

773. (1) $1+\frac{m}{n}x+\frac{m(m-n)}{1.2}\left(\frac{x}{n}\right)^2+\dots.$
$+\frac{m(m-n)\dots(m-\overline{r-1}n)}{1.2.3\,.\,.\,.\,.\,.\,r}\left(\frac{x}{n}\right)^r+\dots.$

(2) $1-\frac{5}{6}x-\frac{5}{72}x^2-\frac{35}{1296}x^3-\dots.$
$-\frac{5.1.7.13\dots(6r-11)}{1.2.3.4\,.\,.\,.\,.\,r}\left(\frac{x}{6}\right)^r-\dots.$

(3) $a^{\frac{3}{4}}+\frac{x}{a^{\frac{1}{4}}}-\frac{1}{6}\frac{x^2}{a^{\frac{5}{4}}}+\frac{5}{54}\frac{x^3}{a^{\frac{9}{4}}}-\dots.$
$+(-1)^{r-1}.\frac{3.1.5.9\dots(4r-7)}{1.2.3.4\,.\,.\,.\,r}.\frac{x^r}{3^r a^{\frac{4r-3}{4}}}+\dots.$

(4) $\left(\frac{2}{3}\right)^{\frac{2}{3}}x^{\frac{2}{3}}-\left(\frac{3}{2}\right)^{\frac{1}{3}}\frac{y}{x^{\frac{1}{3}}}-\frac{3}{8}\left(\frac{3}{2}\right)^{\frac{1}{3}}\frac{y^2}{x^{\frac{4}{3}}}-\dots.$
$-\frac{2.1.4.7\dots(3r-5)}{1.2.3.4\,.\,.\,.\,r}\left(\frac{y}{2}\right)^r\left(\frac{3}{2x}\right)^{\frac{3r-2}{3}}-\dots.$

774. (1) $\frac{1}{c^t}+t.\frac{x}{c^{t+1}}+\frac{t(t+1)}{1.2}\frac{x^2}{c^{t+2}}+\dots.$
$+\frac{t(t+1)\dots(t+r-1)}{1.2\,.\,.\,.\,.\,.\,r}\frac{x^r}{c^{t+r}}+\dots.$

(2) $a^{\frac{1}{s}}-s.\frac{h}{a^{s+1}}+\frac{s(s+1)}{1.2}\frac{h^2}{a^{s+2}}+\dots.$
$+(-1)^r\frac{s(s+1)\dots(s+r-1)}{1.2\,.\,.\,.\,.\,.\,r}\frac{h^r}{a^{s+r}}+\dots.$

(3) $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\frac{35}{128}x^4+\dots.$

(4) $\frac{1}{a^{\frac{2}{3}}}+\frac{1}{3}\frac{x^2}{a^{\frac{8}{3}}}+\frac{2}{9}\frac{x^4}{a^{\frac{14}{3}}}+\frac{14}{81}\frac{x^6}{a^{\frac{20}{3}}}+\ldots.$

(5) $\frac{1}{a^2}\left\{1-2\left(\frac{x}{a}\right)+3\left(\frac{x}{a}\right)^2-4\left(\frac{x}{a}\right)^3+\ldots.\right.$

$\left.+(-1)^r(r+1)\left(\frac{x}{a}\right)^r+\ldots.\right\}.$

775. (1) $a^n+na^{n-1}x\sqrt{-1}-\frac{n(n-1)}{1.2}a^{n-2}x^2+\ldots.$

$+(-1)^{\frac{r}{2}}\frac{n(n-1)\ldots(n-r+1)}{1.2\ldots\ldots r}a^{n-r}x^r+\ldots.$

(2) $b^8-8b^7y\sqrt{-1}-28b^6y^2+56b^5y^3\sqrt{-1}+70b^4y^4$

$-56b^3y^5\sqrt{-1}-28b^2y^6+8by^7\sqrt{-1}+y^8.$

(3) $h^7+7h^6k\sqrt{-1}-21h^5k^2-35h^4k^3\sqrt{-1}+35h^3k^4$

$+21h^2k^5\sqrt{-1}-7hk^6-k^7\sqrt{-1}.$

776. (1) $\frac{1}{(ax)^{\frac{1}{3}}}\left\{1+\frac{1}{3}\left(\frac{x}{a}\right)+\frac{1.4}{3.6}\left(\frac{x}{a}\right)^2+\ldots.\right.$

$\left.+\frac{1.4.7\ldots(3r-2)}{3.6.9\ldots3r}\left(\frac{x}{a}\right)^r+\ldots.\right\}.$

(2) $\frac{1}{a^2}\left\{1+6\left(\frac{x}{a}\right)^{\frac{1}{3}}+\frac{6.7}{1.2}\left(\frac{x}{a}\right)^{\frac{2}{3}}+\ldots.\right.$

$\left.+\frac{6.7\ldots(r+5)}{1.2\ldots r}\left(\frac{x}{a}\right)^{\frac{r}{3}}+\ldots.\right\}.$

(3) $\frac{1}{a}\left\{1-\left(\frac{x}{a}\right)^{\frac{1}{3}}+\left(\frac{x}{a}\right)^{\frac{2}{3}}-\left(\frac{x}{a}\right)+\left(\frac{x}{a}\right)^{\frac{4}{3}}-\ldots.\right\}.$

777. $-126a^4.$

778. $-1400000a^{12}.$

779. $-\frac{5}{4^5}.\frac{b^{18}}{a^{\frac{35}{2}}}.$

780. $-\frac{7\times3^5}{4^4\times4^{\frac{9}{2}}}\left(\frac{y^{\frac{5}{3}}}{a^9.c^{\frac{7}{6}}x^{\frac{9}{2}}}\right).$

781. $715\frac{(by)^4}{(ax)^{14}}.$

782. $-\frac{357}{5^6}.\frac{b^5x^{\frac{28}{5}}}{a^{\frac{22}{5}}}\sqrt{-1}.$

783. $\frac{51060}{7^8}.\frac{t^7}{s^{\frac{54}{7}}}.\left(\sqrt{-1}\right)^{\frac{3}{7}}.$

784. The 1st and 2nd terms respectively.

785. $-252(ax)^{5n}$, and $\frac{1.3.5\ldots(2n-1)}{1.2.3\ldots.n}(2x)^n.$

786. $(-1)^r.\frac{1.6.11\ldots(5r-4)}{1.2.3\ldots\ldots r}.\frac{(2x)}{(5)^r(3a)^{\frac{5r+1}{5}}}$, and

$+\frac{5.1.3\ldots(4r-9)}{1.2.3\ldots.r}.\frac{(3z)^{\frac{r}{2}}}{(4)^r(x^{4r-5}.y^{n-5})^{\frac{1}{4}}}.$

787. $\left(x^{2n}+\frac{1}{x^{2n}}\right)+2n\left(x^{2n-2}+\frac{1}{x^{2n-2}}\right)+\ldots\ldots+\frac{2n(2n-1)\ldots(n+1)}{1.2\ldots\ldots n}$,

and $\left(x^{2n+1}-\frac{1}{x^{2n+1}}\right)-(2n+1)\left(x^{2n-1}+\frac{1}{x^{2n-1}}\right)+\ldots\ldots$

$+(-1)^n\frac{(2n+1)2n\ldots(n+2)}{1.2\ldots\ldots\ldots n}\left(x-\frac{1}{x}\right)$.

791. $\sqrt{\frac{2}{3}}$.

792. $\frac{2-n}{2^{n+1}}$.

793. 0.

794. $\frac{2^{n+1}-1}{n+1}$.

799. $\frac{1.3.5\ldots(2n-1)}{1.2.3\ldots\ldots n}(2^n)$.

800. $\frac{1.3.5\ldots(2n-1)}{1.2.3\ldots(n-1)}\cdot\frac{2^r}{n+1}$.

802. $\frac{n(n^2+11)}{1.2.3}$.

804. $r=\frac{n}{2}-1$.

806. $n=\frac{(2p-1)x\sqrt{P}\pm\sqrt{Px^2+4p(p+1)Qa^2}}{2x\sqrt{P}}$.

809. $1+\frac{b^2h}{a^2+b^2}$ nearly.

810. -1.

811. $\frac{1}{na^{n-1}}$.

812. $\frac{1}{3}$.

813. $\frac{200}{693a}$.

815. 45.

816. $210a^2b^2c^3$.

817. $6ab^5+60ab^3c+60a^3b^2d+60a^3bc^2+30a^4cd$.

818. -79128.

819. $-\frac{m(m-n)(m-2n)}{1.2.3.n^3}\left\{3\frac{bc^2}{a^3}-\frac{m-3n}{n}\cdot\frac{b^3c}{a^4}\right.$

$\left.+\frac{(m-3n)(m-4n)}{4.5.n^2}\cdot\frac{b^5}{a^5}\right\}$.

820. $\frac{n(n-1)(n-2)(n-3)(n-4)}{12}$.

821. $5103000000b^2c^3e^4f$.

822. 4368.

823. The coefficient is

$$\Sigma\left\{\frac{1.2.3\ldots\ldots n}{(1.2\ldots\alpha)(1.2\ldots\beta)(1.2\ldots\gamma)}\text{\&c. } a^{\alpha}b^{\beta}c^{\gamma}\ldots x^{\beta+2\gamma+3\delta+\text{\&c.}},\right\}$$

subject to the conditions $\alpha+\beta+\gamma+\ldots=n$, and $\beta+2\gamma+3\delta+\ldots=6$.

824. $1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\ldots\ldots$

825. $1+2x+3x^2+4x^3+\ldots\ldots$

826. $1+x-\frac{10x^2}{7}+\frac{61x^4}{180}-\ldots\ldots$

827. $1-\frac{2x^3}{3}+\frac{2x^5}{5}+\frac{x^6}{9}-\frac{2x^7}{7}-\frac{2x^8}{15}+\&c.$

828. $\frac{(r+1)(r+2)(r+3)}{1.2.3}.$ 829. $\frac{(n+1)(n+2)}{1.2}.$

830. 35.

831. $$\Big\{1+\frac{n(n-1)}{1.2}+\frac{n(n-1)}{1.2}.\frac{(n-2)(n-3)}{1.2}$$
$$+\frac{n(n-1)(n-2)}{1.2.3}.\frac{(n-3)(n-4)(n-5)}{1.2.3}+\ldots.$$
$$+\frac{n(n-1)\ldots.(n-2r-1)}{(1.2\ldots r)\times(1.2\ldots r)}+\ldots.\Big\}x^n.$$

832. 108073.

INDETERMINATE COEFFICIENTS.

833. $-\frac{2}{x-1}-\frac{10}{x-2}+\frac{18}{x-3}.$

834. $\frac{4}{3(x-2)}-\frac{1}{2(x-1)}+\frac{1}{6(x+1)}.$

835. $\frac{10}{1+2x}-\frac{5}{1+x}-\frac{3}{(1+2x)^2}-\frac{1}{(1+x)^2}.$

836. $\frac{3}{5}-\frac{11}{5^2}x+\frac{7\times 11}{5^3}x^2-\frac{7^2\times 11}{5^4}x^3+\frac{7^3\times 11}{5^5}x^4-\ldots.$

837. $$\frac{a^2+ha+k}{(a-b)(a-c)(x-a)}+\frac{b^2+hb+k}{(b-a)(b-c)(x-b)}$$
$$+\frac{c^2+hc+k}{(c-a)(c-b)(x-c)}.$$

838. $1+3x+4x^2+7x^3+11x^4+18x^5+\ldots.$

839. $\frac{1}{x^3}+\frac{1}{x^2}-\frac{1}{x}+\frac{x-1}{x^2+1}-\frac{1}{(x^2+1)^2}.$

840. $\frac{1}{4a^3(x-a)}-\frac{1}{4a^3(x+a)}-\frac{1}{2a^2(x^2+a^2)}.$

841. $y=x+\frac{x^2}{1.2}+\frac{x^3}{1.2.3}+\frac{x^4}{1.2.3.4}+\ldots.$

842. $x=y-\frac{y^3}{1.2.3}+\frac{y^5}{1.2.3.4.5}-\frac{y^7}{1.2.3.4.5.6.7}+\ldots.$

843. $x=\frac{1}{a}y-\frac{b}{a^3}y^2+\frac{2b^2-ac}{a^5}y^3-\frac{5b^3-5abc+a^2d}{a^7}y^4+\ldots.$

844. $y=1+\frac{x}{3n}-\frac{x^3}{3^4n^3}+\frac{x^4}{3^5n^3}-\ldots.$

845. $y=\sqrt{3}-\frac{x}{6}-\frac{\sqrt{3}x^2}{72}-\frac{x^3}{162}-\frac{19\sqrt{3}.x^4}{6^3.12^2}-\ldots$

846. $\frac{n}{n+1}$, and 1.

847. $\frac{3}{4}-\frac{1}{2}\left(\frac{1}{n+1}+\frac{1}{n+2}\right)$, and $\frac{3}{4}$.

848. $\frac{1}{4}$.

849. $\frac{1}{\sqrt{2}}$.

850. $\frac{1}{4}-\frac{1}{2}\left(\frac{1}{(n+1)(n+2)}\right)$, and $\frac{1}{4}$.

851. 280.

852. 705432.

853. 43758.

SCALES OF NOTATION.

854. 6500445.
855. 212231.
856. 398*e*, 2317, and 8751215.
857. 1456.
858. *t*4*tee*.
859. The radix is 6.
860. The radix is 9.
864. 4112, and 62*te*.
865. $1+2.3+2.3^3+3^5$.
867. $a(1+r)+b(1+nr)+c(1+nr^2)+..+k\left(\frac{r^{n+1}-1}{r-1}\right)$.
868. 248.
870. 6*t*12, 814, and 475*t*968.
871. $\frac{452042}{5550}$ in the senary scale.

PROPERTIES OF NUMBERS.

878. $2^4\times3^3\times5^2\times7\times11^2$, and 21.
883. 72, and 156240.

LOGARITHMS.

MULTIPLICATION BY LOGARITHMS.

886. 146·0347.
887. 14041·29.
888. 23·11337.
889. ·00250075.
890. 12809·97.
891. ·000607593.
892. 17385.
893. 7001600.
894. ·000001712.
895. ·01375935.
896. 103·6616.
897. ·43686.

DIVISION BY LOGARITHMS.

898. 6·2124.
899. ·076.
900. 8·89.
901. ·0275227.
902. 27·997.
903. ·9235.
904. ·222361.
905. ·069927.

INVOLUTION BY LOGARITHMS.

906. 1281·1.
907. 12·5064.
908. ·0000100612.
909. 4·79027.
910. 103006, &c. to 32 places of integers.
911. ·973857.
912. ·99899.

EVOLUTION BY LOGARITHMS.

913. 15·6757.
914. 58·44.
915. 1·08378.
916. 1·00012.
917. ·08075.
918. ·611685.
919. ·702946.
920. ·000 &c. 194368, the number of 0's being 13.
921. ·000 &c. 33968, the number of 0's being 121.

MISCELLANEOUS EXAMPLES IN LOGARITHMS.

922. ·013506.
923. ·00148897.
924. ·30062.
925. ·00000000002567.
926. ·06786.
927. ·94361.
928. 1·34443.
929. ·0000029979.
930. ·035867.
931. ·159133.
932. £69 2*s.* 4¾*d.*

LOGARITHMIC EQUATIONS.

933. $x=\frac{\log b}{\log a}$.
934. $x=1{\cdot}537$.
935. $x=9{\cdot}5868$.
936. $x=\frac{\log c}{b\log a}$.
937. $x=\frac{\log c}{m\log a+n\log b}$.
938. $x=17{\cdot}917$.
939. $x=2{\cdot}342$.
940. $x={\cdot}3716$.
941. 9·6729.

942. $x=\dfrac{c\log m}{c\log a+d\log b}$, $y=\dfrac{d\log m}{c\log a+d\log b}$.

943. $x=m\sqrt{\dfrac{\log c}{m^2\log a+n^2\log b}}$, $y=n\sqrt{\dfrac{\log c}{m^2\log a+n^2\log b}}$.

944. $x=-\infty$, or $-{\cdot}1045$.

945. $x=\dfrac{\log\{b\pm\sqrt{b^2-4}\}-\log 2}{2\log a}$.

946. $x=0$, or 1, or $\left(\dfrac{a}{b}\right)^{\frac{b}{a-b}}$. 947. $x=1{\cdot}242$.

948. $x=\dfrac{\log(a-b)}{\log(a+b)}$.

MENSURATION.

AREAS OF PLANE FIGURES.

1. 23 acres 2 roods 34·56 perches.
2. 1 acre 1 rood 6 perches $10\frac{3}{4}$ yards.
3. $22\frac{1}{3}$ feet. 4. 218·895 yards.
5. 2 roods 21 perches $7\frac{1}{2}$ yards. 6. £5 11*s*. $6\frac{3}{4}$*d*.
7. 10 acres 3 roods 8 perches $12\frac{1}{2}$ yards.
8. 105·6 yards. 9. 1 square foot.
10. 42 rows.
11. 2 acres 1 rood 8 perches 12 yards.
12. 83·53 yards. 13. $82\frac{2}{3}$ yards.
14. $22\frac{13}{96}$ feet. 15. $128\frac{1}{4}$ yards.
16. 387·1 feet. 17. 584·57 feet.
18. 817·63 feet. 19. 1086·4 feet.
20. 15·708 feet. 21. 1·5915 chains.
22. 113·0976 feet. 23. 3·1416 feet.
24. 4·63386 feet. 25. $1622\frac{1}{2}$ yards.
26. 1776·82 inches. 27. 509·38 feet.
28. 88·802 feet. 29. 157·08 feet.
30. 502·656 feet. 31. 273·875 feet.
32. £2 9*s*. $9\frac{3}{4}$*d*. 33. $6\frac{1}{2}$*d*. nearly.
34. 14·69 inches. 35. 22·42 feet.
36. 315 yards. 37. 25 yards.
38. 801·1 feet. 39. 177637 feet.
40. 183·783 feet. 41. 56 yards.

SURFACES AND CONTENTS OF SOLIDS.

42. 284·924 gallons.
43. 127·2348 cubic feet.
44. 1164·924 cubic feet, and 744 square feet.
45. 130·98 cubic feet, and 260·94 square feet.
46. 34·36 square feet.
47. 1187 cubic feet.
48. 110·187 square feet, and 76·759 cubic feet.
49. 589·068 cubic feet.
50. 144 square feet.
51. 76·737 square feet, and 54·126 cubic feet.
52. 2880 cubic feet.
53. 27137 cubic feet.
54. 254·469 square feet, and 381·704 cubic feet.
55. 1256·64 square feet, and 5236 cubic feet.
56. 213·629 square feet.
57. $\frac{1}{4}$ surface, and $\frac{5}{32}$ volume.
58. $\frac{1}{2}$ radius, and $\frac{2}{27}$ volume.
59. ·00374 of the whole volume.
60. 9149976000 miles.

61. The number of tons is equal to 54051, etc. to 16 places of integers.

62. $\frac{3}{8}$ of a solid foot.
63. The ratio is 4 : 1.
64. $932\frac{4}{7}$ bales.
65. 897·42 gallons.
66. $16\frac{4}{7}$ feet.
67. $2666\frac{2}{3}$ cubic feet.
68. 5·289 rods.
69. £6 15*s*. 2*d*. nearly.
70. £6 10*s*.
71. £28 4*s*.
72. £29 7*s*. 6*d*.
73. £2 8*s*. 3*d*.

MEASUREMENT OF SHOT, SHELLS, AND POWDER.

74. 243 lbs.
75. 11·24 inches.
76. 72·14 lbs.
77. 5·8 inches.
78. 5·78 lbs.
79. 2·65 inches.
80. $172\frac{4}{5}$ lbs.
81. 14·42 inches.
82. 12·72 lbs.
83. 8·3 inches.
84. 94·248 lbs.
85. 10·61 inches.
86. 4·46 inches.
87. 422·231 lbs.
88. 17·68 inches.
89. 11·588 inches.

TRIGONOMETRY.

TRIGONOMETRICAL FORMULÆ.

1. $48^\circ\ 45'$, and $26^\circ\ 15'$; $54\frac{1}{6}g$, and $29\frac{1}{6}g$.
2. $\frac{3\times 3}{2\times 5}$, $\frac{3\times 3^2}{2\times 5^2}$, and $\frac{3\times 3^3}{2\times 5^3}$.
3. ·99484, 1·0035, ·31416, ·94248, and ·3927.
4. $57^\circ\ 17'\ 44''\cdot 8$, $28^\circ\ 38'\ 52''\cdot 4$, $85^\circ\ 56'\ 37''\cdot 2$, $19^\circ\ 5'\ 54''\cdot 9$, $38^\circ\ 11'\ 49''\cdot 8$, and $47^\circ\ 44'\ 47''\cdot 2$.
5. $\sin\frac{A}{2} = \sqrt{\frac{r^2 - r\cos A}{2}}$.
6. $r^2 \sin 3A = 3r^2 \sin A - 4\sin^3 A$.
7. $\cot A = \frac{r^3 \sin A}{\cos A \,.\, \tan^2 A}$.
8. $\tan 3A \,.\, \tan A = \frac{r^2 \sin 3A \,.\, \sin A}{\cos 3A \,.\, \cos A}$.
9. $\tan^2 2A - \tan^2 A = \frac{r^4 \sin 3A \,.\, \sin A}{\cos^2 2A \,.\, \cos^2 A}$.
10. $r^2 + \cos 2A \,.\, \cos 2B = \cos^2(A+B) + \cos^2(A-B)$.

---

51. $\frac{\sqrt{2-\sqrt{2+\sqrt{3}}}}{2}$, $\frac{\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}}{4}$, and $\frac{\sqrt{2-\sqrt{2}}}{2}$.
52. $\frac{\sqrt{5}-1+\sqrt{30+6\sqrt{5}}}{8}$, $\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$, and $\frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2}$.
53. $\sqrt{\frac{3-\sqrt{5}}{5+\sqrt{5}}}$, $\sqrt{2}-1$, and $\frac{2+\sqrt{3}}{2+\sqrt{2-\sqrt{3}}}$.
54. $\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$, $2-\sqrt{3}$, and 1.
55. $\sqrt{4-2\sqrt{2}}$, $\sqrt{\frac{10+2\sqrt{5}}{5}}$, and $-\sqrt{2}$.
56. $\frac{2}{3}\sqrt{3}$, $\sqrt{\frac{10-2\sqrt{5}}{5}}$, and 2.
57. $\frac{2\sqrt{2}-\sqrt{3}-1}{2\sqrt{2}}$, $\frac{2-\sqrt{2-\sqrt{2}}}{2}$, and $\frac{3}{2}$.
58. $\frac{\sqrt{5}-1}{2}$, $\sqrt{2-\sqrt{2}}$, and $\sqrt{3}$.

90. $A=0$, or 60°, or 120°. 91. $A=0$, or 30°.

92. $A=45$, or 90°, or 135°. 93. $\sin A=\frac{\pm\sqrt{5}-1}{2}$.

94. $A=45^\circ$, or $\tan^{-1} 3$.

95. $A=15^\circ$, or 30°, or 45°, or 75°.

96. $A=90^\circ$, or $\frac{60}{n-1}$. 97. $A=60^\circ$, or 120°.

98. $A=0$, or 60°. 99. $A=15^\circ$, or 75°.

100. $\sin A=\frac{\pm\sqrt{m^2+4n^2}-m}{2n}$.

101. $\sin A=\frac{\sqrt{1+2m}\pm\sqrt{1-2m}}{2}$.

102. $A=45^\circ$, or 120°. 103. $A=18^\circ$, or 54°.

104. $x=45^\circ$. 105. $x=45^\circ$, or 135°.

106. $\cos x=\cos\alpha\pm 1$. 107. $x=2\alpha$, or $\pi-2\alpha$.

114. $x=\frac{1}{2}$, or ± 1. 115. $\sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2}$.

116. $\tan\frac{\phi}{2}.\tan\left(45-\frac{\psi}{2}\right)$. 117. a.

118. $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$. 119. $\frac{\sin\alpha.\sin\phi}{1-\sin\alpha.\cos\phi}$.

120. $\frac{\tan^2\frac{\alpha}{2}.\sin 2\phi}{1+\tan^2\frac{\alpha}{2}.\cos 2\phi}$. 121. $\frac{1}{m}=\frac{1}{2}\left(\cos^{\frac{2}{3}}\theta+\sin^{\frac{2}{3}}\theta\right)^{\frac{3}{2}}$.

122. $m^{\frac{2}{3}}.n^{\frac{2}{3}}\left(m^{\frac{2}{3}}+n^{\frac{2}{3}}\right)=1$.

123. $\tan\frac{\alpha}{2}.\tan\frac{\gamma}{2}=\tan\frac{\beta}{2}$.

124. $(a-m)(n-a)b^2=(b-n)(m-b)a^2$.

PROPERTIES OF PLANE FIGURES.

159. $r=2\sqrt{\frac{2}{7}}$, and $R=\frac{45}{4\sqrt{14}}$.

160. 216. 161. 30.

162. One side is $r\sqrt{\frac{m+n+1}{(m+n-1)(1+m-n)(1+n-m)}}$, and the sine of the opposite angle is

$$\sqrt{(m+n+1)(m+n-1)(1+m-n)(1+n-m)}.$$

163. $c=\frac{2\text{ area}}{a\sin B}$.

164. $a=2R\sin A$, $b=2R\sin B$, and $C=2R\sin C$.

165. The ratios are $3:5:7$; the angle $=120^\circ$.

167. $\dfrac{\sqrt{5+\sqrt{5}}}{\sqrt{10}}$.

168. For the pentagons $\dfrac{3+\sqrt{5}}{8}$; for the octagons $\dfrac{\sqrt{2}+1}{2\sqrt{2}}$.

169. $\dfrac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{\sqrt{2}}$.

170. $\sqrt{3}:\sqrt{2}$.

171. $3\sqrt{3}:(2\sqrt{2})^2$.

174. $5+\sqrt{5}:2$.

176. $a\cot\dfrac{\pi}{2n}$.

177. $\dfrac{a^2}{2}\cdot\dfrac{5+2\sqrt{5}}{\sqrt{10-2\sqrt{5}}}$.

179. $r_0=\dfrac{r_1r_2r_3}{r_1r_2+r_1r_3+r_2r_3}$, $S=\sqrt{r_0r_1r_2r_3}$.

180. $2\sqrt{a^2-c^2}+2c\,.\cos^{-1}\left(-\dfrac{c}{a}\right)$.

181. $a\operatorname{cosec}^2\dfrac{\pi}{2n}$.

182. The ratios are $\sqrt[4]{27}:\sqrt[4]{16}:\sqrt[4]{12}$.

SOLUTION OF PLANE TRIANGLES.

183. $B\ =71^\circ\,46'$.
$BC=135{\cdot}167$.
$AC=410{\cdot}31$.

184. $B\ =47^\circ\,25'$.
$AB=437{\cdot}958$.
$AC=322{\cdot}466$.

185. $A\ =64^\circ\,40'\,47''$.
$BC=1115{\cdot}45$.
$AC=527{\cdot}754$.

186. $A\ =33^\circ\,6'\,25''$.
$AB=219{\cdot}661$.
$BC=119{\cdot}979$.

187. $BC=957{\cdot}039$.
$A\ =88^\circ\,33'\,49''$.
$B\ =1^\circ\,26'\,11''$.

188. $A\ =5^\circ\,45'\,7''$.
$B\ =84^\circ\,14'\,53''$.
$AC=3544{\cdot}06$.

189. $A\ =54^\circ\,5'\,1''$.
$B\ =35^\circ\,54'\,59''$.
$AB=721{\cdot}1$.

190. $C\ =100^\circ\,22'\,45''$.
$AB=1758{\cdot}78$.
$BC=1337{\cdot}18$.

191. $A\ =127^\circ\,45'$.
$AB=115{\cdot}905$.
$BC=407{\cdot}401$.

192. $C\ =55^\circ\,55'\,43''$.
$A\ =35^\circ\,50'\,17''$.
$BC=372{\cdot}5$.

193. $B\ =54^\circ\,10'\,56''$, or $125^\circ\,49'\,4''$.
$C\ =73^\circ\,30'\,4''$, or $1^\circ\,51'\,56''$.
$AB=393{\cdot}755$, or $13{\cdot}3668$.

194. $C\ =27^\circ\,5'\,42''$, or $152^\circ\,54'\,18''$.
$B\ =144^\circ\,34'\,43''$, or $18^\circ\,46'\,7''$.
$AC=1590{\cdot}1$, or $882{\cdot}72$.

195. $B\ =23^\circ\,8'\,33''$.
$A\ =32^\circ\,17'\,27''$.
$BC=171{\cdot}268$.

196. $C\ =90^\circ$.
$B\ =36^\circ\,46'$.
$AC=311{\cdot}878$.

197. C = 76° 44′ 55″.
B = 53° 59′ 5″.
BC = 331·633.

198. A = 157° 3′ 31″.
C = 7° 43′ 15″.
AC = 105·4075.

199. A = 67° 58′ 51″.
B = 59° 7′ 4″.
C = 52° 54′ 5″.

200. A = 51° 8′ 4″.
B = 68° 18′ 50″.
C = 60° 33′ 6″.

HEIGHTS AND DISTANCES.

201. 212·099 feet.
202. 732·21 feet.
203. 75·6735, 43·4045, and 61·988 yards.
204. 228·631 yards.
205. 537·187 yards.
206. 49·643 feet.
207. 56·43 feet.
208. 48° 22′.
209. 115·47, and 9·503 yards.
210. 345·534 yards.
211. 39·65 yards.
212. 10·201 yards.
213. 910·82, and 845·54 yards.
214. 101·891 feet.
215. 3513·325 yards.
216. 48·633 feet.
217. 100·137 yards.
218. 82·804 yards.
219. 578·6 yards.
220. 413·8, and 707·6 yards.
221. 6·143, and 8·792 miles.
222. 6·3397 miles.
223. N. 76° 56′ E., and 13·938 miles an hour.
224. 2·36 miles.
225. 1° 42′.
226. 7937·86 miles.
227. SA = 1469·76, SB = 1650·10, and SC = 1155·91 yards.
228. SA = 1664·23, SB = 1493·45, and SC = 2030·92 yards.
229. SA = 896·34, SB = 1172, and SC = 1506·34 yards.
230. SA = 23·556, SB = 58·74, and SC = 23·347 yards.
231. 234·6 feet.
232. 200·21 feet.

EXPANSIONS, SERIES, ETC.

233. $\frac{1}{32}\cos 6\theta+\frac{3}{16}\cos 4\theta+\frac{15}{32}\cos 2\theta+\frac{5}{16}.$

234. $$(\cos\theta)^{2n}=\frac{1}{2^{n-1}}\left\{\cos 2n\theta+2n\cos(2n-2)\theta+\dots\dots\frac{1}{2}\,\frac{2n(2n-1)\dots.(n+1)}{1.2.3\dots.n}\right\}.$$

235. $$(\cos\theta)^{2n+1}=\frac{1}{2^{n}}\left\{\cos(2n+1)\theta+(2n+1)\cos(2n-1)\theta+\dots.+\frac{(2n+1).2n\dots(n+2)}{1.2\dots n}\cos\theta\right\}.$$

236. $$(\sin\theta)^{4m}=\frac{1}{2^{4m-1}}\left\{\cos 4m\theta-4m\,.\cos(4m-2)\theta\right.$$
$$\left.+\dots\frac{1}{2}\frac{4m(4m-1)\dots\dots(2m+1)}{1.2\dots\dots2m}\right\}.$$

237. $$(\sin\theta)^{4m+1}=\frac{1}{2^{4m}}\left\{\sin(4m+1)\theta-(4m+1)\sin(4m-1)\theta\right.$$
$$\left.+\dots+\frac{(4m+1).4m\dots\dots(2m+2)}{1.2\dots\dots2m}\sin\theta\right\}.$$

238. $$(\sin\theta)^{4m+2}=-\frac{1}{2^{4m+1}}\left\{\cos(4m+2)\theta-(4m+2)\cos 4m\theta\right.$$
$$\left.+\dots\dots-\frac{1}{2}\frac{(4m+2)(4m+1)\dots\dots(2m+2)}{1.2\dots\dots(2m+1)}\right\}.$$

239. $$(\sin\theta)^{4m+3}=-\frac{1}{2^{4m+2}}\left\{\sin(4m+3)\theta-(4m+3)\sin(4m+1)\theta\right.$$
$$\left.+\dots\dots-\frac{(4m+3)(4m+2)\dots\dots(2m+3)}{1.2\dots\dots(2m+1)}\sin\theta\right\}.$$

240. $\sin 4\theta=4\sin\theta\cos^3\theta-4\cos\theta\,.\sin^3\theta.$
$\sin 9\theta=9\sin\theta\,.\cos^8\theta-84\sin^3\theta\,.\cos^6\theta$
$+126\sin^5\theta\,.\cos^4\theta-36\sin^7\theta\,.\cos^2\theta+\sin^9\theta.$

241. $\cos 5\theta=\cos^5\theta-10\sin^2\theta\,.\cos^3\theta+5\sin^4\theta\,.\cos\theta,$
$\cos 6\theta=\cos^6\theta-15\sin^2\theta\,.\cos^4\theta+15\sin^4\theta\,.\cos^2\theta-\sin^6\theta,$
$\cos 7\theta=\cos^7\theta-21\sin^2\theta.\cos^5\theta+35\sin^4\theta.\cos^3\theta-7\sin^6\theta.\cos\theta.$

242. $\tan 3\theta=\dfrac{3\tan\theta}{1-3\tan^2\theta},$

$$\tan 8\theta=\frac{8\tan\theta-56\tan^3\theta+56\tan^5\theta-8\tan^7\theta}{1-28\tan^2\theta+70\tan^4\theta-28\tan^6\theta+\tan^8\theta}.$$

249. $\dfrac{\sin(n-1)x\,.\cos nx}{\sin\frac{x}{2}}.$

250. $\dfrac{x^{n+1}\cos(n-1)\theta-x^n\cos n\theta-x\cos\theta+1}{2(1-x\cos\theta)}.$

251. $\dfrac{\sin(n+1)a\,.\sin na}{\sin\frac{a}{2}}.$

252. 0.

253. $2\cot 2\theta-\dfrac{1}{2^{n-1}}\cot\dfrac{\theta}{2^{n-1}}.$

254. $\cot\theta-2^n\cot 2^n\theta.$

255. $\cot\dfrac{a}{2}-\cot 2^{n-1}a.$

256. $\dfrac{1-\cos\theta\,.\cos\phi}{1+\cos^2\theta-2\cos\theta\,.\cos\phi}.$

257. $\dfrac{\sin(n+1)\phi.\cos(2\theta+n\phi)}{\sin\frac{\phi}{2}}.$

258. $\theta.$

259. $\frac{n}{2}+\frac{\sin n\alpha\,.\cos(2\theta+\overline{n-1\alpha})}{2\sin\alpha}$.

260. $\frac{\sin n\theta-\sqrt{2}\,.\sin\overline{n+1}\theta+\sqrt{2^{n+1}}\,.\sin\theta}{3\sqrt{2^n}-2\sqrt{2^{n+1}}\,.\cos\theta}$.

261. $\frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}\cdot(\cos\theta+\sqrt{-1}\,.\sin\theta)^{\frac{n+1}{2}}$, and $\sqrt{-1}\,.\frac{(\cos\theta+\sqrt{-1}\sin\theta)^{\frac{1}{2}}}{2\sin\frac{\theta}{2}}$.

262. $\frac{\pi}{6}$.

264. $x=n\sin a+\frac{n^2}{2}\sin 2a+\frac{n^3}{3}\sin 3a+\ldots.$

265. $\theta=m\cos\phi-\frac{m^2}{2}\sin 2\phi+\frac{m^3}{3}\cos 3\phi+\frac{m^4}{4}\sin 4\phi+\ldots.$

269. $\cdot 6931471$.

SOLUTION OF EQUATIONS.

272. $x=1\pm\sqrt{3}$, or -2.

273. $x=14\cos 20^\circ$, or $-14\cos 40^\circ$, or $-14\cos 18^\circ$.

274. $x=-2\cos 20^\circ$, or $2\cos 40^\circ$, or $2\cos 80^\circ$.

275. $x=2$, or -1. 276. $x=-1$, or $\frac{1\pm\sqrt{-3}}{2}$.

277. $x=\frac{\pm\sqrt{2}\pm\sqrt{-2}}{2}$.

278. $x=1$, or $\frac{\sqrt{5}-1\pm\sqrt{-10-2\sqrt{5}}}{4}$,

or $\frac{-\sqrt{5}-1\pm\sqrt{-10+2\sqrt{5}}}{4}$.

279. $x=\pm\sqrt{-1}$, or $\frac{\pm\sqrt{3}\pm\sqrt{-1}}{2}$.

280. $x=\pm 1$, or $\pm\sqrt{-1}$, or $\frac{\pm\sqrt{2}\pm\sqrt{-2}}{2}$.

281. $x=\cos 20^\circ\pm\sqrt{-1}\sin 20^\circ$, or $-\cos 40^\circ\pm\sqrt{-1}\,.\sin 40^\circ$,
or $-\cos 80^\circ\pm\sqrt{-1}\,.\sin 80^\circ$.

SUBSIDIARY ANGLES.

282. $x=a^2\sec^2\phi$. 283. $x=2\sqrt{a}.\sin\frac{\phi}{2}$.

284. $x=\frac{\sqrt{a-b}}{a+b}\sec^2\phi$. 285. $\cos x=\frac{\cos A}{\sin\phi}\cdot\sin(B-\phi)$.

286. $x=\frac{(a-b)\cos\frac{C}{2}}{\cos\phi}$. 287. $\sin(x+\phi)=\frac{c.\cos\phi}{a}$.

STATICS.

FORCES ACTING ON ONE POINT AND IN THE SAME PLANE.

1. $\sqrt{6}:2:\sqrt{3}-1$. 2. 105°, 120°, and 135°.

3. 15·237 lbs. the magnitude, and 28° 50′ the inclination to the direction of the 1st force.

4. 6·889 the magnitude, and 102° 16′ the inclination to the direction of the 1st force.

5. 62° 43′ 13″. 6. 4·63.

8. 125° 41′. 9. The ratio is $\sqrt{2}:1$.

11. 150°.

15. The resultant is 5; and the point of its application on the rod, is at a distance of 3·0217 feet from the end.

16. $\tan^{-1}\left(\frac{W}{P}\right)$. 17. The inclination of the strings to each other =60°.

18. Each of the strings CA, CB is inclined at an angle of 30° to the horizon.

19. BC = ·638 AB; BAC = 32° 32′ 3″·2; ABC = 24° 55′ 53″·5.

20. The cosines of the angles which the strings make with each other are respectively $-\frac{1}{8}$, $-\frac{9}{16}$, and $-\frac{3}{4}$.

22. Pressure = one of the weights.

23. The ratio is 11 : 3. 24. The plane is vertical.

25. The ratio is $\sin\beta:\sin\alpha$. 26. PAQ = 32° 32′.

27. $\tan PCA=\frac{P}{Q}$. 29. $\frac{W\sin\alpha}{\sin(\alpha+\beta)}$, and $\frac{W\sin\beta}{\sin(\alpha+\beta)}$.

FORCES ACTING IN ONE PLANE, NOT THROUGH THE SAME POINT.

30. AC represents this fourth force. 32. $2\tan\theta=\frac{\sin(\beta-\alpha)}{\sin\alpha . \sin\beta}$.

33. Tension $=\frac{1}{\sqrt{3}}$W; pressure on two upper tacks $=\frac{\sqrt{3}}{2}$W; pressure on two lower tacks $=0$.

35. There is equilibrium for all positions of AB, when $Q=\frac{1}{2}W$.

37. CA $=4$ feet. 38. AP $=1\cdot838$ times radius.

39. Tension $=\frac{\sqrt{3}}{4}$W. 40. Tension $=\frac{36}{125}$W.

41. BC.tan ACD $=$ AC.tan BCE, and ED $=$ AC.cos ACD $+$ BC.cos BCE.

42. Vertical pressure $=$ W; force $=$ horizontal pressure $=\frac{W}{2}$.

43. 90°, or $\cos^{-1}\sqrt{\frac{7}{8}}$. 44. $\frac{91}{100}$ weight of the 3 beams.

45. $\frac{\cot\alpha}{2}$.W. 46. 30°.

47. $6\frac{1}{2}$ lbs. nearly.

48. $\cos^{-1}\sqrt[3]{\frac{b}{a}}$. 49. $4\frac{1}{2}$ feet.

50. $\sin\theta=\sqrt{\frac{2m}{a}}-1$, or $\theta=90^\circ$, $x=\sqrt{2ma}$, or m.

51. $\tan^{-1}.\frac{ac+bc-a^2}{ac+bc-b^2}$.

52. $\frac{(W-P+Q)b}{2W}$, and $\frac{(W-Q+P)b}{2W}$.

53. $b-n+\sqrt{n^2+b^2}$. 54. The ratio is $\sqrt{2}:\sqrt{3}$.

55. $3\frac{10}{21}$ feet. 56. 90 lbs.

57. $\tan^{-1}\frac{a^2+2ab}{b^2}$.

58. The equations to determine θ and ϕ are

$$\tan\theta=\frac{6\sin\phi-1}{6\cos\phi},\ \text{and}\ \tan\phi=\frac{1-\sin\theta}{1-\cos\theta}.$$

59. 30°. 60. $4a^3:b(2a^2-c^2)$.

62. At the point where the weight 7 hangs.

63. $a-b\tan\alpha:a+b\tan\alpha$. 64. $53^\circ\ 27'\ 23''$.

65. $\sqrt{2} : 1$, and 120°.

66. At $\frac{a}{\sqrt{2}}$ from the ground.

67. The weight is 216 lbs.; the distance from the end $5\frac{19}{34}$ feet.

68. At $\frac{2}{3}na$ from the end.

69. 20 lbs.

70. 60°.

FORCES WHICH DO NOT ACT IN THE SAME PLANE.

71. Each distance $= 1$.

72. Each pressure $= \frac{1}{3}$ weight.

73. Each tension $= \frac{1}{3}$ weight.

74. $\cos\theta = \frac{W}{nP}$.

THE CENTRE OF GRAVITY.

76. $\frac{\sqrt{a^4 + b^4 - 2a^2b^2\cos\alpha}}{2(a+b)}$.

77. $\frac{1}{3}$ difference of the perpendicular distances of the opposite angles from the same diagonal.

78. $\frac{c}{4} \cdot \frac{a^4 - 4ab^3 + 3b^4}{a^4 - a^3b - ab^3 + b^4}$.

79. $\frac{1}{4}$ sum of the altitudes.

80. $\frac{c}{4} \cdot \frac{a^2 + 2ab + 3b^2}{a^2 + ab + b^2}$.

81. $\frac{a^6 - 3a^2b^4 + 2b^6}{3m(a^4 - b^4)}$.

82. $\frac{a^3 + a^2b + ab^2 + b^3}{a^2 + ab + b^2}$.

83. $\frac{3}{8}\,\frac{4a^3 + 6a^2c + 4ac^2 + c^3}{3a^2 + 3ac + c^2}$.

84. The shorter side is horizontal.

85. $\cot^2\theta = \frac{8a^2 - 6b^2}{3b^2}$.

86. Slant height of the cone $= 2$ radius of the hemisphere.

87. The altitude of the paraboloid $= \sqrt{\frac{3}{2}}$ the radius of hemisphere.

88. $AD = AB\sqrt{\frac{17}{2}}$.

89. $\frac{180°}{n}$.

90. 16° 26′.

91. $9m\cot^2\alpha$.

92. $\frac{\pi a^2b^2}{3\sqrt{a^2 + b^2}}$.

93. $\frac{\text{diameter}}{3\pi}$.

94. One volume $= 3$ times the other.

95. The ratio is $5 : 7$.

96. $\frac{\pi a}{3}(b^2 + bc + c^2)$.

MACHINES.

98. The ratio is 19 : 24. 99. $\tan\theta = \dfrac{a(P-Q)}{b(P+Q+W)}$.

100. The ratio is $\sqrt{7}-1 : 2$.

101. His gain per cent. is $\dfrac{50(2m+1)}{m^2+m}$.

102. The true weight is 6 lbs.; the arms are 2 ft. 3 in., and 1 ft. 6 in.

103. The longer arm is inclined at an angle of 48° 22′ below the horizon, the shorter at an angle of 18° 22′ above.

104. 1st, 4·851 lbs.; 2nd, 3·647 lbs.

105. 2, 8, 14, 20, &c. inches successively.

106. $16\frac{8}{13}$, $15\frac{3}{7}$, $14\frac{2}{5}$, $13\frac{1}{2}$, $12\frac{12}{17}$, 12 inches successively.

107. $3\frac{22}{37}$ inches. 108. 29·1146 lbs.

109. $1234\frac{2}{7}$ lbs. 110. 264 lbs.

111. 120°. 112. 7 moveable pulleys.

114. The ratio is 64 : 63. 115. 100 lbs.

116. 756 lbs. 117. The ratio is 1 : 247.

118. 13·4 lbs.

119. 44° 25′ the plane to the horizon; 11° 32′ the force to the plane.

120. 20° 16′ the plane to the horizon, 57° 12′ the power to the plane.

121. $\sin^{-1}\dfrac{PR}{Wr}$. 122. ·396 lb.

123. 678·58 lbs. 124. 3·3157 yards.

FRICTION.

125. The limits are $W \cdot \dfrac{\sin\alpha + \mu\cos\alpha}{\sin\beta + \mu\cos\beta}$, and $W \cdot \dfrac{\sin\alpha - \mu\cos\alpha}{\sin\beta - \mu\cos\beta}$.

126. The least force $= 5$ lbs., the angle $= 30°$.

127. 90°. 128. $\dfrac{1}{\sqrt{3}}$.

129. W must not be less than P. In the limiting value, $W = P$.

130. $\tan\theta = \dfrac{1+\mu\mu'}{2\mu'(\mu+1)}$, pressure on wall $= \dfrac{\mu' W}{1+\mu\mu'}$, pressure on plane $= \dfrac{W}{1+\mu\mu'}$.

131. He will ascend $\frac{5}{7}$th of the ladder's height.

132. $\sin\theta = \dfrac{8\mu'(1+\mu)}{3(1+\mu\mu')}$.

DYNAMICS.

THE COLLISION OR IMPACT OF BODIES.

1. Common velocity after impact $=6\frac{17}{19}$ feet, velocity lost by $A=1\frac{2}{19}$, velocity gained by $B=1\frac{17}{19}$.
2. $A : B :: 37 : 11$.
3. A's velocity before impact $=38\frac{1}{4}$ feet.
4. $u=-4\frac{5}{11}$ feet, $v=3\cdot9\dot{4}\dot{5}$ feet.
5. Common velocity after impact $=4\frac{1}{2}$ feet.
6. Velocity lost by $A=\cdot08a$, velocity gained by $B=\cdot12a$.
7. $u : v :: 29 : 36$. 8. $u=-\frac{1}{2}a$, $v=\frac{5}{2}a$.
9. The number of bodies $=7$. 10. $e=\frac{3}{5}$.
11. Angle of incidence $=30°$.
12. Ball required $=\frac{A+B}{2}\left\{e\pm\sqrt{e^2-\frac{4AB}{(A+B)^2}}\right\}$.

The limits between which e must lie are $\frac{2\sqrt{AB}}{A+B}$ and 1.

13. $v=\sqrt{\frac{A}{B}}\cdot a$ is the limit required.
15. $\tan\theta=\sqrt{\frac{e^3}{1+e+e^2}}$, θ being measured from a diameter passing through the point of projection.
17. $\frac{A}{B}=\frac{5}{3}$, $\frac{B}{C}=2$, $\frac{C}{D}=3$. If n be the number of balls,

$$\frac{A}{B}=\frac{n+1}{n-1},\ \frac{B}{C}=\frac{n}{n-2},\ \frac{C}{D}=\frac{n-1}{n-3},\ \ldots$$

18. $PO=\frac{B^2}{B^2-A^2}(2a-h)$, where O is the point of meeting, $PQ=2a$, and the given velocity $=\sqrt{2gh}$; A, B the balls.
19. Velocity of plane $=2(1-e)\sqrt{\frac{ag}{2e-e^2}}$, where $2a=$ given distance, and g measures the force of gravity.
20. Space $=6\cdot62$ feet; time $=0''\cdot86$.
23. Distance required $=123$ feet.
24. $\tan\theta=\frac{1}{e}\tan\alpha$, and $v=\frac{\sin\alpha}{\sin\theta}\cdot a$, θ being the $\angle$ of reflexion.
27. $e=\frac{A}{B}=\frac{1}{6}$.

UNIFORMLY ACCELERATED MOTION AND GRAVITY.

31. $s = 1948{\cdot}1$ feet, $v = 354{\cdot}2$ feet.
32. $t = 15''\frac{1}{2}$, $s = 3883$ feet.
33. $v = 170{\cdot}22$ feet, momentum $= 10{\cdot}64$ tons.
34. The spaces are to one another as 29 : 13.
35. $s' = 275{\cdot}42$ feet. 36. $s' = 225{\cdot}4$ feet.
37. $s = 15970$ feet. 38. $s = 63{\cdot}6$ feet.
39. Velocity of projection $= \sqrt{2g(a + \frac{1}{2}g\tau^2)}$, whole time $= 2\sqrt{\frac{2}{g}(a + \frac{1}{2}g\tau^2)}$, where $2\tau =$ given time, and $a =$ distance from the point of projection to the given point.
40. $V = 28{\cdot}4$ feet.
41. $120{\cdot}4$ feet from the bottom of steeple.
42. $t = \frac{1}{g}\{u \pm \sqrt{u^2 + 2gh}\}$, where $h =$ given height and $u =$ given velocity of balloon.
43. $\frac{g}{g'} = \left\{\left(1 + \frac{mT}{4a}\right) \pm \frac{1}{4a}\sqrt{m^2T^2 + 8amT}\right\}^2$, g, g' measuring the force of gravity at the first and second stations respectively.
44. The time required is $1''{\cdot}42$ nearly.
45. $f = \frac{P - Q}{p - q}$, $V = \frac{Q(2p - 1) - P(2q - 1)}{2(p - q)}$.
46. $1915{\cdot}9$ feet.
47. $s = 123{\cdot}228$ feet; $t = 5''{\cdot}461$; $v = 118{\cdot}67$ feet.
48. $7306{\cdot}2$ feet.
49. The upper extremity of the line is $10{\cdot}195$ feet from the top of the plane.
50. 32 feet. 51. 19° 28′ 16″.
52. $2\frac{1}{3}$ oz.
53. If α, β be the inclinations of the first and second planes, x, c their respective lengths, then will $x = c \cdot \frac{\sin\beta}{\sin\alpha . \cos^2(\alpha + \beta)}$.
54. Side $= \frac{a}{16}$, whole time $= \frac{1}{\sqrt{ga\sqrt{3}}}\left\{\frac{5a}{\sqrt{8}} + 2d\right\}$.
57. Distance from bottom of plane $= \frac{P + W}{P} \cdot \frac{hl}{h + l}$.

The time $= \frac{(P + W)l}{P}\sqrt{\frac{2hP}{(Pl - Wh)(h + l)g}}$.

58. $\frac{1}{9}l$, $\frac{1}{3}l$, $\frac{5}{9}l$. 59. By geometrical construction.

60. At a distance $l\cdot\frac{4n-1}{4n}$ from the bottom. 61. $l=\frac{V^2\left(\frac{4}{9}-e^2\right)}{2(1-e^2)g\sin\alpha}$.

65. $x^2-y^2=a^2$, where $2a$ = distance between the two given points, and x is measured from the middle point between them.

66. The inclination of the required diameter to the vertical $=85°\ 5'$.

67. The abscissa of C measured along OB $=\cdot9428\times$ OB.

68. $A=2\cdot732B$. 71. $V=10$ ft.

72. Velocity of sound $=1173\cdot8$ ft.

73. $\frac{\text{vertical side}}{\text{horizontal side}}=\frac{3}{4}$; $11\cdot852$ ft., and $22\cdot31$ ft.

78. $\frac{52\cdot8}{1-e^2}$ ft. 79. $2''\cdot024$, and $2''\cdot545$.

80. $v^2=2g\left(a+\frac{P-Q}{P+Q}x\right)$, $x=\sqrt{2ga}\,.\,t+\frac{P-Q}{P+Q}\cdot\frac{gt^2}{2}$, where t'' is the time of P's falling through x feet after having fallen through a feet.

81. $9\cdot00931$, and $8\cdot99069$. 82. $\frac{W}{5\cdot05}$.

83. $s=203\cdot68$ ft., $v=46\cdot92$ ft. 85. $\theta=\cos^{-1}\left(\frac{1}{2e}\right)$.

ON MOTION UPON A CURVE AND ON THE SIMPLE PENDULUM.

86. $\frac{1}{4}\left\{\frac{3g}{2}(9a+6\sqrt{3}\,b+8c)\right\}^{\frac{1}{2}}$, where a, b, c denote the lengths of the planes A, B, C respectively.

88. $2\cdot4462$ ins. and 1739.52 ins.

89. $52\cdot036$ ins. 90. $l=\left(\frac{n}{n\pm1}\right)^2.L$.

91. $1''\cdot8398$. 92. $2\frac{1}{4}$ miles nearly.

93. Shortened by $\cdot00906$ in. 94. $33\frac{1}{4}$ miles.

95. Radius of the earth $=\frac{ht_1^2}{t_1^2-t^2}$, where h = depth below the surface, t, t_1 the times on the surface and below respectively.

96. 23 hrs. $56'\ 3''\cdot98$, and $\cdot2133$ ins.

97. $32\cdot075$ ft., and $32\cdot256$ ft.

98. $568''$. 100. $3\frac{1}{3}$ miles.

ON THE MOTION OF PROJECTILES IN A NON-RESISTING MEDIUM.

101. Latus rectum = 11219 feet nearly, coordinates of focus are $\left.\begin{array}{l} x=9825{\cdot}65 \\ y=5609{\cdot}5 \end{array}\right\}$ feet.

102. $R=55{\cdot}916$ ft., $T=0''{\cdot}965$, $H=3{\cdot}746$ ft.

103. $V=126{\cdot}87$ ft., $H=125$ ft., $T=5''{\cdot}5736$.

104. $V=2595{\cdot}4$ ft., and $\alpha=1^\circ\ 48'\ 30''$.

105. Ratio required $=\dfrac{x}{x\sin\alpha-y\cos\alpha}$, x, y being coordinates of any point in parabola described; $\alpha=45^\circ$.

106. $\alpha=74^\circ\ 33'\ 47''$; $V=250{\cdot}45$ ft., $H=905{\cdot}4$ ft.

107. $\alpha=53^\circ\ 8'$; $\alpha=75^\circ\ 58'$ in the two cases respectively.

108. $\alpha=56^\circ\ 19'$.

109. $V=165{\cdot}48$ ft., $T=9''{\cdot}778$.

110. $R=179{\cdot}869$ ft.

111. Height above the plane = 247 ft., velocity required = 81·2 ft., and direction is given by $\phi=157^\circ\ 30'$.

112. $1 : 4\cos^2\alpha$. 113. $\alpha=60^\circ$.

115. α is found from the equation $\sin(\alpha-\theta)=\pm\sqrt{\dfrac{a}{2h}}\sin 2\theta$, where $y=(x+a)\,.\tan\theta$ is the equation to plane.

116. Distance from vertex $=\dfrac{8V^2}{3g}\sin(30^\circ+\alpha)\cos\alpha$.

117. Horizontal range = 128 ft., time of flight = $5''{\cdot}12$.

118. Height of tower $=\dfrac{4h\cos\alpha\,.\cos\beta\,.\cot(\alpha+\beta)}{\sin(\alpha+\beta)}$, where α, β are the angles of projection.

119. Latus rectum $=4h\dfrac{\cos^2\alpha}{\sin\iota}$.

120. $V=64{\cdot}542$ ft.; time $=2''{\cdot}8355$.

121. 503·2 ft.

122. 21·637 ft. from the base of the house; time from the eaves $=1''{\cdot}1$.

123. $AD=AC\times 1{\cdot}7071$; time required $=\sqrt{\dfrac{AC}{g}}\,.\times 2{\cdot}765$.

124. The path is a parabola, whose equation involves $\tan\alpha=\dfrac{2Au+Bv}{Bv\sqrt{3}}$, and $V^2\cos^2\alpha=\dfrac{3B^2v^2}{4(A+B)^2}$.

125. Height after $3''=1917{\cdot}088$ ft., $H=7337{\cdot}7$ ft.

126. The locus is a circle whose equation is $\left(y+\dfrac{1}{2}gt^2\right)^2+x^2=V^2t^2$, the point of projection being the origin.

127. The locus is an ellipse whose equation is $x^2 = 4(hy - y^2)$, the minor axis $= h$, the major axis $= 2h$.

128. The locus is a parabola expressed by $r = \frac{2h}{1 + \cos\theta}$, where $\theta = \frac{\pi}{2} - \iota$.

129. $R = \left(\frac{4}{3}\right)^{\frac{3}{2}} . l\epsilon$, where $l =$ length of inclined plane, and ϵ the elasticity.

130. Distance required $= 4h\epsilon(1 + \epsilon)\sin\theta$, where $h =$ given height, $\theta =$ angle of incidence.

131. $\epsilon = \cdot 5$, and one ball is double of the other.

132. $\tan\alpha = \frac{1}{2}(3\tan\iota + \cot\iota)$.

133. $R = d \times \cdot 3248$.

135. $R = \frac{2h\sin 2\alpha}{1 - \epsilon}$.

136. 2·118 miles.

137. 1·554 lbs.

138. [illegible] lbs. 3 oz.; time $= 12'' \frac{2}{3}$.

THE END.

Printed by Richard and John E. Taylor, Red Lion Court, Fleet Street.

ERRATA.

Page 19, Ex. 42, *for* $-2e^x$ *read* $-e^x$.
—— 22, — 87, *for* $\frac{a}{b}$ *read* $\frac{a}{d}$.
—— 27, — 193, *for* ab^2 *read* ab^3.
—— 27, — 212, *for* $-3-2\sqrt{-2}$ *read* $-3-2\sqrt{2}$.
—— 28, — 261, *for* $-\frac{3}{2}a^2b^{\frac{1}{4}}$ *read* $-\frac{3}{2}a^2b^{\frac{1}{2}}$.
—— 34, — 395, *for* $-5x$ *read* $-5z$.
—— 57, — 806, *for* qth *read* $(p+2)$th.
—— 60, — 867, *for* kx^n *read* kr^n.
—— 63, — 913, *for* square *read* cube.
—— 66, — 18, *for* half *read* one-fourth of.
—— 86, — 117, *for* tan 2θ *read* tan θ.
—— 93, — 220, *for* AB = 650 *read* AD = 650.
—— 93, — 228, line 30, *for* the angle ASC *read* the angles ASC and ASB.
—— 107, — 90, *for* e, e' *read* ϱ, ϱ'.
—— 123, — 54, *for* 30° *read* 45°, and *for* 45° *read* 30°.
—— 143, — 100, *for* 0″·05 *read* 0′·05.
—— 150, — 103, *for* $\cdot 538\dot{5}8024691\dot{1}3$ *read* $\cdot 538\dot{5}8024691\dot{3}$.
—— 151, — 156, *for* ·1175019 *read* ·94.
—— 153, — 252, *for* 5*s.* *read* 15*s.*
—— 153, — 259, *for* £598 *read* £698.
—— 158, — 136, *for* $\frac{27}{2}\sqrt{3}$ *read* $\frac{1}{2}\sqrt{3}$.
—— 158, — 150, *for* $\frac{1}{2}\sqrt[12]{a^7}$ *read* $\frac{1}{3}\sqrt[12]{a^7}$.
—— 158, — 162, *for* $a^{-\frac{11}{5}}$ *read* $a^{-\frac{17}{5}}$.
—— 158, — 163, *for* (x^2+y^2) *read* $(x^2+y^2)^{\frac{1}{3}}$.
—— 159, — 188, *for* $\sqrt{x^2+1}+\sqrt{x^2-1}$ *read* $2x^2$.
—— 159, — 192, *for* $\frac{b^{\frac{1}{n}}}{a^{\frac{1}{m}}}$ *read* $\frac{b^{\frac{1}{m}}}{a^{\frac{1}{n}}}$.
—— 159, — 213, *for* $1-\sqrt{-3}$ *read* $1+\sqrt{-3}$.
—— 160, — 272, *for* $\frac{8+3\sqrt{2}+2\sqrt{3}+\sqrt{6}}{8}$ *read* $\frac{4+3\sqrt{2}-2\sqrt{\ }-\sqrt{6}}{4}$.
—— 161, — 305, *for* $x=9$ *read* $x=-9$.
—— 164, — 440, *for* $\pm\frac{\sqrt{9493}-35}{12}$ *read* $\frac{\pm\sqrt{34297}-35}{48}$.
—— 167, — 505, *for* 4·25, $-2\frac{66}{67}$ *read* 2·25, $-2\frac{527}{536}$.
—— 167, — 506, *for* $x=8$, $-\cdot 5$ *read* $x=5$, $\frac{3}{4}$,

and *for* $y=6$, $-2\cdot 5$ *read* $y=3$, $-\frac{5}{4}$.

—— 171, — 582, *for* 3 yards *read* −30 yards.
—— 171, — 585, *for* 21 *read* 21 and 40.
—— 175, — 743, *for* 12859 *read* 26059.
—— 180, — 890, *for* 12809·97 *read* 12811·35.
—— 181, — 907, *for* 12·5064 *read* 125·064.
—— 181, — 924, *for* ·30062 *read* ·019875.
—— 181, — 925, *for* ·0000000002567 *read* ·000016025.
—— 184, — 56, *for* $\frac{2}{3}\sqrt{3}$ *read* 2; and *for* 2 *read* $\frac{2}{3}\sqrt{3}$.
—— 185, — 90, *dele* or 120°.
—— 186, — 190, *for* BC *read* AC.
—— 193, — 107, *for* $3\frac{22}{37}$ *read* $1\frac{11}{37}$.